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Novel Interval Valued T-Spherical Fuzzy Mclaurin Symmetric Mean Operators and Their Applications in Multi-Attribute Group Decision Making Problems

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ABSTRACT

Educational institutes play a significant role to build up a nation and developing society. Education is an important factor that can help a man to judge his destiny, and shape the coming future. This article aims to extend the concepts of interval-valued T-spherical fuzzy (IVTSF) set (IVTSFS) based on Maclaurin symmetric mean (MSM) operators. The main concentration of this manuscript is to study the interrelationship among any number of IVTSF numbers (IVTSFNs) by the use of Maclaurin symmetric mean (MSM) operators. We explore and develop IVTSF Maclaurin symmetric mean (IVTSFMSM) operator, IVTSF weighted MSM (IVTSFWMSM) operator, IVTSF dual MSM (IVTSFDMSM) operator, IVTSF weighted dual MSM (IVTSFWDMSM) operator. By using these examined operators, some special cases of the discovered operators are also established and their properties are examined. In addition, a procedure for handling multi-attribute group decision-making (MAGDM) techniques based on MSM operators in the IVTSF setting. A demonstrative example to check the applicability of the MSM operators of IVTSFSs is presented which established the selection of applicants. To show the supremacy of the newly established MSM operators, a comprehensive comparative study is designed numerically.

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1. Introduction

To describe the belongingness of objects to certain phenomena under uncertainty and vagueness, the theory of intuitionistic fuzzy set (IFS) was investigated by Atanassov (1986), Additionally, the theory of interval- Valued (IV) IFS was deliberated by Atanassov (1999) which is the extension of the fuzzy set (FS) (Zadeh, 1965). Additionally, the theory of interval-valued (IVFS) was deliberated by Bustince, H et al. (2009). IFS is a generalized form of FS to manage vague data in day-to-day life issues where the degree of membership (DM) and the degree of non-membership (DNM) are denoted by m_i and d_i. Some recent work on IVIFS can be found in (Garg & Rani, 2019a), (Garg & Kumar, 2019), (Burillo & Bustince, 1996), (Grzegorzewski, 2004), (Garg & Rani 2019b). To handle such kind of information, a less restricted fuzzy framework of Pythagorean FS (PyFS) was developed by Yager (2013). Some recent work on the theory of IVPyFS can be seen in (Peng & Li, 2019), (Garg, 2016), (Garg, 2018). Likewise, IVIFS and IVPyFS also face applicability issues as some duplets $\left([m^{\ell}, m^{u}], [d^{\ell}, d^{u}]\right)$ cannot be sorted by IVIFS or IVPyFS. Due to the fact, a relatively more flexible frame of q-rung orthopair FS (QROFS) was developed by Yager (2016) that allows the q-power of the DM and DNM in the range of 0 to 1. the theory of interval-valued QROFS (IVQROFS) was deliberated by Joshi et al. (2018). This framework of IVQROFS removes the previously existing barriers by introducing a variable parameter $q \in \mathbb{Z}^{+}$. Due to this parameter q every duplet can be regarded to lie in the frame of IVQROFS. Some recent work on IVQROFSs can be seen in (Garg, 2021), (Ali & Mahmood, 2020), (Garg et al., 2021a).

Cuong (2013) observed that the description of uncertain events using the DM and the DNM is not sufficient and the opinion of remaining degree of abstain denoted by (DA)and the refusal degree expressed by (DR) have also a key role in the description modeling of the human observation. Cuong presented the notion of picture fuzzy set (PFS). The theory of IV picture fuzzy set (IVPFS) was deliberated by Khalil et al. (2019). A great amount of research on IVPFS and its applications is studied in (Liu et al., 2019), (Wei et al., 2019). In numerous practical situations, the theory of IVPFS is not applicable. Due to this fact, the theory of spherical FS (SFS) and TSFS was developed by Mahmood et al. (2019). The notion of SFS and TSFS significantly enlarge the range for assigning the DM, DNM, and DA but still, some triplets are uncategorized which leads to Mahmood et al. The theory of IV spherical fuzzy (IVSF) set (IVSFS) and IVTSFS were deliberated by K. Ullah et al. (2019). to develop the notion of IVTSFS and IVTSFS that provides limitless flexibility for the assigning of the DM, DNM, DA, etc. An IVTSFS has an associated variable parameter $q \in \mathbb{Z}^{+}$ that can categorize every triplet as an IVTSF number (IVTSFN). The IVTSFS is a novel addition to the study of fuzzy sets and its generalization and has gotten serious attention recently. Recent studies on the aggregation operators (AOs) of the IVTSFSs can be seen in (Mahmood et al., 2020), (Garg et al., 2018), (Garg et al., 2021b), (Ullah et al., 2019), while studies on the information measures of the IVTSFSs can be seen in (Hwang & Rhee, 2004), (Gorzałczany, 1987). Some studies on the IVTSF graphs and IVTSF soft sets can be viewed in (Kalathian et al., 2020), (Ashtiani et al., 2009).

To find the interrelationships among any fuzzy information, the MSM operators are considered among the significant AOs. The theory of the MSM operator was initiated by Maclaurin (1729) as an aggregation tool for non-negative real numbers. This concept was extended to generalized MSM operators of two variables by DeTemple and Robertson (1979). The concept of MSM operators was greatly utilized in various fuzzy settings to comply with information under uncertainties. MSM operators of IVIFSs are investigated in the problems of multi-attribute decision making (MADM) by Qin and Liu (2014). Wei et al. (2018) extended the notion of MSM operators to the IVPyF environment which covers a wide range of information in the MADM process. By observing the restricted range of the MSM operators of the IVIFSs and IVPyFS, Wang et al. (2019) developed the MSM operators for IVQROFSs where the ability to aggregate the uncertain information has increased sufficiently. It also handles the information in the context of IVIFSs, IVPyFSs, and IVq-ROPFS. If we observe the theory of MSM operators developed in (DeTemple & Robertson, 1979), (Qin & Liu, 2014), (Wei et al., 2018), (Wang et al., 2019), it discusses only two aspects of uncertain information by using the DM and DNM where the abstinence and refusal information is lost. To reduce the information loss and to incorporate the abstinence and refusal degrees of the information into account while aggregating information, we aim to develop MSM operators in the frame of IVTSFS. By doing so, we incorporate the four aspects of the uncertain information with the help of the DM, DNM, DA, and DR associated by a varying parameter $q \in \mathbb{Z}^{+}$ that ensures a large range for the various degrees. Some key features of the manuscript are discussed below:

1. Investigation of the notion of IVTSFMSM operator in the frame of IVTSFS using the DM, DNM, DA, and DR.
2. A study of the characteristics of the IVTSMSM operators.
3. Investigating the superiority of the IVTSMSM operators over the previously defined MSM operators of IVIFSs, IVPyFSs, and IVq- QROFSs theoretically and with the help of examples.
4. Setting up a MAGDM procedure based on IVTSFMSM operators.

The summary of this paper is as follows: In section 2, we recall the idea of IVTSFS, and their fundamental laws followed by an introduction to the notion of MSM operator. In section 3, we develop IVTSFMSM operator, IVTSFWMSM operator followed by an investigative study. In section 4, the idea of MSM operator is further extended to IVTSFDMSM and IVTSFWDMSM operator. Moreover, we also study some special cases of our proposed methodology. In section 5, it is proved that the IVTSFMSM operator is a generalized form of the previously defined MSM operators with the help of some restrictions. In section 6, a MADM procedure based on IVTSFMSM operators is developed followed by a thorough numerical example to show the applicability of our defined operators. A comparative investigation of our defined theory is established with previously obtained MSM operators in section 7 where the advantages of the currently defined MSM operators over the MSM operators of IVIFSs, IVq-QROFSs, and IVPyFSs are proposed. In the end, the whole paper is summarized in conclusion and some future work is discussed in section 8.

2. Preliminaries

Firstly, we study the notion of IVTSFS and its fundamental terms and notions. For further development of this article we expressed the idea of MSM operators. Throughout this article, the symbol Y represents the universal set. and the triplet (m, a, d) denotes the DM, DA, and DF. Throughout this paper $\varrho = 1, 2, 3 \dots n$ and $\mathfrak{x} = 1,2,3, \dots n$ denote the indexing terms. Throughout this paper $(y) = [m^{\ell}(y), m^{u}(y)]$ $, a(y) = [a^{\ell}(y), a^{u}(y)], d(y) = [d^{\ell}(y), d^{u}(y)]$. We discuss some basic definitions of IVTSFS (Ullah et al., 2019), MSM operator and DMSM operator (Maclaurin, 1729).

Definition 1: (Ullah et al., 2019) A IVTSFS P is elaborated by:

$$P = \left\{ \left(y, \left(m(y), a(y), d(y) \right) \right) : y \in Y \right\}$$

with a condition that $0 \leq (m^q)^u(y) + (a^q)^u(y) + (d^q)^u(y) \leq 1, q \in \mathbb{Z}^+$. Here $(y) = [m^{\ell}(y), m^{u}(y)]$ $, a(y) = [a^{\ell}(y), a^{u}(y)], d(y) = [d^{\ell}(y), d^{u}(y)]$ The expression $r(y) = [r^{\ell}(y), r^{u}(y)] = \left[\sqrt[q]{1 - \left((m^u)^q(y) + (a^u)^q(y) + (d^u)^q(y) \right)}, \sqrt[q]{1 - \left((m^{\ell})^q(y) + (a^{\ell})^q(y) + (d^{\ell})^q(y) \right)} \right]$ is termed as DR. We call $P = (m, a, d) = \left([m^{\ell}(y), m^{u}(y)], [a^{\ell}(y), a^{u}(y)], [d^{\ell}(y), d^{u}(y)] \right)$ a IVTSF number (IVTSFN).

Definition 2: (Ullah et al., 2019) Let $P = (m, a, d), P_1 = (m_1, a_1, d_1)$ and $P_2 = (m_2, a_2, d_2)$ be three IVTSFNs and $\gamma > 0$. Then

1. $P_1 \oplus P_2 =$
$$\left(\left[\sqrt[q]{\left(m_1^{\ell}\right)^q + \left(m_2^{\ell}\right)^q - \left(m_1^{\ell}\right)^q \left(m_2^{\ell}\right)^q}, \sqrt[q]{(m_1^u)^q + (m_2^u)^q - (m_1^u)^q (m_2^u)^q} \right] \left[a_1^{\ell} a_2^{\ell}, a_1^u a_2^u \right], \left[d_1^{\ell} d_2^{\ell}, d_1^u d_2^u \right] \right)$$
2. $$P_1 \otimes P_2 = \begin{pmatrix} \left[m_1^{\ell} m_2^{\ell}, m_1^u m_2^u \right], \left[\sqrt[q]{\left(a_1^{\ell}\right)^q + \left(a_2^{\ell}\right)^q - \left(a_1^{\ell}\right)^q \left(a_2^{\ell}\right)^q}, \sqrt[q]{(a_1^u)^q + (a_2^u)^q - (a_1^u)^q (a_2^u)^q} \right], \\ \left[\sqrt[q]{\left(d_1^{\ell}\right)^q + \left(d_2^{\ell}\right)^q - \left(d_1^{\ell}\right)^q \left(d_2^{\ell}\right)^q}, \sqrt[q]{(d_1^u)^q + (d_2^u)^q - (d_1^u)^q (d_2^u)^q} \right] \end{pmatrix}$$
3. $\gamma P = \left(\left[\sqrt[q]{1 - (1 - (m^{\ell})^q)^{\gamma}}, \sqrt[q]{1 - (1 - (m^u)^q)^{\gamma}} \right], \left[\left(a^{\ell}\right)^q, (a^u)^q \right], \left[\left(d^{\ell}\right)^q, (d^u)^q \right] \right)$
4. $(P)^{\gamma} =$
$$\left(\left[\left(m^{\ell}\right)^{\gamma}, (m^u)^{\gamma} \right], \left[\sqrt[q]{1 - (1 - (a^{\ell})^q)^{\gamma}}, \sqrt[q]{1 - (1 - (a^u)^q)^{\gamma}} \right], \left[\sqrt[q]{1 - (1 - (d^{\ell})^q)^{\gamma}}, \sqrt[q]{1 - (1 - (d^u)^q)^{\gamma}} \right] \right)$$
5. $(P^c) = \left([d^{\ell}, d^u], [a^{\ell}, a^u], [m^{\ell}, m^u] \right)$

Definition 3: Consider $P = (m, a, d) = \left([m^{\ell}(y), m^{u}(y)], [a^{\ell}(y), a^{u}(y)], [d^{\ell}(y), d^{u}(y)] \right)$ be any IVTSFN. Then score function and accuracy function is expressed as:

$$\dot{\mathrm{S}}(P) = \frac{\left(m^{\ell}\right)^q \left(1 - \left(a^{\ell}\right)^q - \left(d^{\ell}\right)^q\right) + (m^u)^q (1 - (a^u)^q - (d^u)^q)}{3}, \dot{\mathrm{S}}(P) \in [0,1]$$

$$\hat{\mathrm{E}}(P) = (m^q + a^q + d^q), \hat{\mathrm{E}}(P) \in [0,1]$$

Based on the above-defined two rules, we categorize any two IVTSFNs $P_1 = (m_1, a_1, d_1)$ and $P_2 = (m_2, a_2, d_2)$ then the score value can be obtained as $\dot{\mathrm{S}}(P_1) = \frac{\left(m_1^{\ell}\right)^q \left(1 - \left(a_1^{\ell}\right)^q - \left(d_1^{\ell}\right)^q\right) + (m_1^u)^q (1 - (a_1^u)^q - (d_1^u)^q)}{3}$ and $\dot{\mathrm{S}}(P_2) = \frac{\left(m_2^{\ell}\right)^q \left(1 - \left(a_2^{\ell}\right)^q - \left(d_2^{\ell}\right)^q\right) + (m_2^u)^q (1 - (a_2^u)^q - (d_2^u)^q)}{3}$ and accuracy value can be calculated as $\hat{\mathrm{E}}(P_1) = m_1^q + a_1^q + d_1^q$ and $\hat{\mathrm{E}}(P_2) = m_2^q + a_2^q + d_2^q$. Then

1. If $\dot{\mathrm{S}}(P_1) < \dot{\mathrm{S}}(P_2)$ then $P_1 < P_2$
2. If $\dot{\mathrm{S}}(P_1) = \dot{\mathrm{S}}(P_2)$ then

1) If $\hat{E}(P_1) < \hat{E}(P_2)$ then $P_1 < P_2$
2) If $\hat{E}(P_1) = \hat{E}(P_2)$ then $P_1 = P_2$

Definition 4: (Maclaurin, 1729) Let P_ϱ be the collection of positive numbers. then the MSM operator is elaborated by:

$$MSM^K(P_1, P_2, \dots, P_n) = \left(\frac{\sum_{1 \le i_1 \le \cdots \le i_ᵵ \le n} \prod_{\varrho=1}^{ᵵ} P_{i_\varrho}}{C_n^{ᵵ}}\right)^{\frac{1}{ᵵ}}$$

where C_n^k represents the binominal coefficients and $(i_1, i_2, i_3, \dots, i_ᵵ)$ denote all the k-tuples. MSM operator satisfies the following conditions:

1. $MSM^K(0,0,\dots,0) = 0$.
2. $MSM^K(P, P, \dots, P) = P$.
3. $MSM^K(P_1, P_2, \dots, P_n) \le MSM^K(Q_1, Q_2, \dots, Q_n)$ if $P_i \le Q_i \ \forall\, i$.
4. $min\{P_i\} \le MSM^K(P_1, P_2, \dots, P_n) \le max\{P_i\}$

Definition 5: (Maclaurin, 1729) Let P_ϱ be the collection of positive numbers then the DMSM operator is elaborated by:

$$DMSM^K(P_1, P_2, \dots, P_n) = \frac{1}{ᵵ}\left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_ᵵ \le n}} \left(\sum_{\varrho=1}^{ᵵ} P_{i_\varrho}\right)^{\frac{1}{C_n^{ᵵ}}}\right)$$

Where binomial coefficient is denoted by C_n^k with $(i_1, i_2, \dots, i_ᵵ)$ represent the k-tuple combination of $(1,2,\dots,n)$.

3. IVTSFMSM and IVTSFWMSM Operators

In this study, we combine the idea of IVMSM and IVWMSM operators with IVTSFSs to investigate the idea of IVTSFMSM and IVTSFWMSM operators and discussed their different properties. Moreover, the special cases of the elaborated operators are also discussed.

Definition 6: Let $P_\varrho = (m_\varrho, a_\varrho, d_\varrho)$ be the collection of IVTSFNs. Here $m_\varrho = [m_\varrho^\ell, m_\varrho^u]$, $a_\varrho = [a_\varrho^\ell, a_\varrho^u]$ and $d_\varrho = [d_\varrho^\ell, d_\varrho^u]$. Then the IVTSFMSM operator is elaborated by:

$$IVTSFMSM^{ᵵ}(P_1, P_2, P_3, \dots, P_n) = \left(\frac{\bigoplus_{1 \le i_1 \le \cdots i_ᵵ \le n} \left(\bigotimes_{\varrho=1}^{ᵵ} P_{i_\varrho}\right)}{C_n^{ᵵ}}\right)^{\frac{1}{ᵵ}}$$

Where binomial coefficient is denoted by C_n^k and $(i_1, i_2, \dots, i_ᵵ)$ represent the k-tuple combination of $(1,2,\dots,n)$.

Theorem 1: Let $P_\varrho = (m_\varrho, a_\varrho, d_\varrho)$ be the collection of IVTSFNs. Here $m_\varrho = [m_\varrho^\ell, m_\varrho^u]$, $a_\varrho = [a_\varrho^\ell, a_\varrho^u]$ and $d_\varrho = [d_\varrho^\ell, d_\varrho^u]$. Then by using the idea of IVTSFMSM operators, we obtain:

$IVTSFMSM\,(P_1, P_2, P_3, \ldots P_n)$

$$
= \left(\begin{array}{c}
\left[\left(\sqrt[q]{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_{\mathfrak{x}} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} m^{\ell}_{i_\varrho} \right)^{q} \right) \right)^{\frac{1}{C_n^{\mathfrak{x}}}}} \right)^{\frac{1}{\mathfrak{x}}}, \left(\sqrt[q]{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_{\mathfrak{x}} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} m^{u}_{i_\varrho} \right)^{q} \right) \right)^{\frac{1}{C_n^{\mathfrak{x}}}}} \right)^{\frac{1}{\mathfrak{x}}} \right], \\
\left[\sqrt[q]{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_{\mathfrak{x}} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(a^{\ell}_{i_\varrho} \right)^{q} \right) \right) \right) \right)^{\frac{1}{C_n^{\mathfrak{x}}}} \right)^{\frac{1}{\mathfrak{x}}}}, \sqrt[q]{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_{\mathfrak{x}} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(a^{u}_{i_\varrho} \right)^{q} \right) \right) \right) \right)^{\frac{1}{C_n^{\mathfrak{x}}}} \right)^{\frac{1}{\mathfrak{x}}}} \right], \\
\left[\sqrt[q]{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_{\mathfrak{x}} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(d^{\ell}_{i_\varrho} \right)^{q} \right) \right) \right) \right)^{\frac{1}{C_n^{\mathfrak{x}}}} \right)^{\frac{1}{\mathfrak{x}}}}, \sqrt[q]{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_{\mathfrak{x}} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(d^{u}_{i_\varrho} \right)^{q} \right) \right) \right) \right)^{\frac{1}{C_n^{\mathfrak{x}}}} \right)^{\frac{1}{\mathfrak{x}}}} \right]
\end{array} \right)
$$

Proof: By using Definition (6) we have:

$$
\bigotimes_{\varrho=1}^{\mathfrak{x}} P_{i_\varrho} = \left(\begin{array}{c}
\left[\prod_{\varrho=1}^{\mathfrak{x}} m_{i_\varrho}^{\ell}, \prod_{\varrho=1}^{\mathfrak{x}} m_{i_\varrho}^{u} \right], \left[\sqrt[q]{1 - \prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(a_{i_\varrho}^{\ell} \right)^{q} \right)}, \sqrt[q]{1 - \prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(a_{i_\varrho}^{u} \right)^{q} \right)} \right], \\
\left[\sqrt[q]{1 - \prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(d_{i_\varrho}^{\ell} \right)^{q} \right)}, \sqrt[q]{1 - \prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(d_{i_\varrho}^{u} \right)^{q} \right)} \right]
\end{array} \right)
$$

and

$$
\bigoplus_{\substack{1 \le i_1 \le \cdots \\ < i_{\mathfrak{x}} \le n}} \left(\bigotimes_{\varrho=1}^{\mathfrak{x}} P_{i_\varrho} \right) = \left(\begin{array}{c}
\left[\sqrt[q]{1 - \prod_{\substack{1 \le i_1 \le \cdots \\ < i_{\mathfrak{x}} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} m^{\ell}_{i_\varrho} \right)^{q} \right)}, \sqrt[q]{1 - \prod_{\substack{1 \le i_1 \le \cdots \\ < i_{\mathfrak{x}} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} m^{u}_{i_\varrho} \right)^{q} \right)} \right], \\
\left[\prod_{\substack{1 \le i_1 \le \cdots \\ < i_{\mathfrak{x}} \le n}} \sqrt[q]{1 - \prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(a_{i_\varrho}^{\ell} \right)^{q} \right)}, \prod_{\substack{1 \le i_1 \le \cdots \\ < i_{\mathfrak{x}} \le n}} \sqrt[q]{1 - \prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(a_{i_\varrho}^{u} \right)^{q} \right)} \right], \\
\left[\prod_{\substack{1 \le i_1 \le \cdots \\ < i_{\mathfrak{x}} \le n}} \sqrt[q]{1 - \prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(d^{\ell}_{i_\varrho} \right)^{q} \right)}, \prod_{\substack{1 \le i_1 \le \cdots \\ < i_{\mathfrak{x}} \le n}} \sqrt[q]{1 - \prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(d^{u}_{i_\varrho} \right)^{q} \right)} \right]
\end{array} \right)
$$

$$\frac{1}{C_n^{\mathfrak{x}}}\bigoplus_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(\bigotimes_{\varrho=1}^{\mathfrak{x}}P_{i_\varrho}\right)=\left(\begin{array}{c}\left[\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}m_{i_\varrho}^{\ell}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}},\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}m_{i_\varrho}^{u}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right],\\ \left[\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\sqrt[q]{1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right)}\right)^{\frac{1}{C_n^{\mathfrak{x}}}},\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\sqrt[q]{1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right],\\ \left[\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\sqrt[q]{1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)}\right)^{\frac{1}{C_n^{\mathfrak{x}}}},\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\sqrt[q]{1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right]\end{array}\right)$$

Therefore

$IV\text{TSFMSM}(P_1,P_2,\dots,P_n)=$

$$\left(\begin{array}{c}\left[\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}m_{i_\varrho}^{\ell}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}},\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}m_{i_\varrho}^{u}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}\right],\\ \left[\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}},\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}}\right],\\ \left[\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}},\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}}\right]\end{array}\right)$$

Moreover, the ideas of idempotency, monotonicity, and boundedness are developed.

Property 1: Let $p_\varrho=\left(m_\varrho,a_\varrho,d_\varrho\right)$ be the collection of IVTSFNs. Here $(y)=[m^{\ell}(y),m^{u}(y)]$,$a(y)=[a^{\ell}(y),a^{u}(y)]$, $d(y)=[d^{\ell}(y),d^{u}(y)]$. If $P_\varrho=P$ then $IVTSFMSM(P_1,P_2,P_3,\dots,P_n)=P$.

Proof: We know that $P_\varrho=\left(m_\varrho,a_\varrho,d_\varrho\right)$ and $P=(m_P,a_P,d_P)$. Here $m_P=\left[m_P^{\ell},m_P^{u}\right]$, $a_P=\left[a_P^{\ell},a_P^{u}\right]$, $d_P=\left[d_P^{\ell},d_P^{u}\right]$ then by using Theorem (1) we obtain:

$IV\mathrm{TSFMSM}(P, P, \ldots, P) =$

$$
\left(\begin{array}{c}
\left[\left(\sqrt[q]{1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} m_P^{\ell}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}},\left(\sqrt[q]{1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} m_P^{u}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}\right],\\
\left[\left(\sqrt[q]{1-\left(1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_P^{\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)}\right)^{\frac{1}{\mathfrak{x}}},\left(\sqrt[q]{1-\left(1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_P^{u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)}\right)^{\frac{1}{\mathfrak{x}}}\right],\\
\left[\left(\sqrt[q]{1-\left(1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_P^{\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)}\right)^{\frac{1}{\mathfrak{x}}},\left(\sqrt[q]{1-\left(1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_P^{u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)}\right)^{\frac{1}{\mathfrak{x}}}\right]
\end{array}\right)
$$

$$
=\left(\begin{array}{c}
\left[\left(\sqrt[q]{1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(m_P^{\ell}\right)^{q\mathfrak{x}}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}},\left(\sqrt[q]{1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(m_P^{u}\right)^{q\mathfrak{x}}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}\right],\\
\left[\left(\sqrt[q]{1-\left(1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(1-\left(a_P^{\ell}\right)^{q}\right)^{\mathfrak{x}}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)}\right)^{\frac{1}{\mathfrak{x}}},\left(\sqrt[q]{1-\left(1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(1-\left(a_P^{u}\right)^{q}\right)^{\mathfrak{x}}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)}\right)^{\frac{1}{\mathfrak{x}}}\right],\\
\left[\left(\sqrt[q]{1-\left(1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(1-\left(d_P^{\ell}\right)^{q}\right)^{\mathfrak{x}}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)}\right)^{\frac{1}{\mathfrak{x}}},\left(\sqrt[q]{1-\left(1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(1-\left(d_P^{u}\right)^{q}\right)^{\mathfrak{x}}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)}\right)^{\frac{1}{\mathfrak{x}}}\right]
\end{array}\right)
$$

$$
=\left(\begin{array}{c}
\left[\left(\sqrt[q]{1-\left(\left(1-\left(m_P^{\ell}\right)^{q\mathfrak{x}}\right)^{C_n^{\mathfrak{x}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}},\left(\sqrt[q]{1-\left(\left(1-\left(m_P^{u}\right)^{q\mathfrak{x}}\right)^{C_n^{\mathfrak{x}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}\right],\\
\left[\left(\sqrt[q]{1-\left(1-\left(\left(1-\left(1-\left(a_P^{\ell}\right)^{q}\right)^{\mathfrak{x}}\right)^{C_n^{\mathfrak{x}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)}\right)^{\frac{1}{\mathfrak{x}}},\left(\sqrt[q]{1-\left(1-\left(\left(1-\left(1-\left(a_P^{u}\right)^{q}\right)^{\mathfrak{x}}\right)^{C_n^{\mathfrak{x}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)}\right)^{\frac{1}{\mathfrak{x}}}\right],\\
\left[\left(\sqrt[q]{1-\left(1-\left(\left(1-\left(1-\left(d_P^{\ell}\right)^{q}\right)^{\mathfrak{x}}\right)^{C_n^{\mathfrak{x}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)}\right)^{\frac{1}{\mathfrak{x}}},\left(\sqrt[q]{1-\left(1-\left(\left(1-\left(1-\left(d_P^{u}\right)^{q}\right)^{\mathfrak{x}}\right)^{C_n^{\mathfrak{x}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)}\right)^{\frac{1}{\mathfrak{x}}}\right]
\end{array}\right)
$$

$$=\left(\begin{array}{c}\left[\left(\sqrt[q]{1-\left(\left(1-\left(m_P^{\ell}\right)^{q\mathfrak{x}}\right)\right)}\right)^{\frac{1}{\mathfrak{x}}},\left(\sqrt[q]{1-\left(\left(1-\left(m_P^{u}\right)^{q\mathfrak{x}}\right)\right)}\right)^{\frac{1}{\mathfrak{x}}}\right],\\ \left[\sqrt[q]{1-\left(1-\left(\left(1-\left(1-\left(a_P^{\ell}\right)^{q}\right)^{\mathfrak{x}}\right)\right)\right)^{\frac{1}{\mathfrak{x}}}},\sqrt[q]{1-\left(1-\left(\left(1-\left(1-\left(a_P^{u}\right)^{q}\right)^{\mathfrak{x}}\right)\right)\right)^{\frac{1}{\mathfrak{x}}}}\right],\\ \left[\sqrt[q]{1-\left(1-\left(\left(1-\left(1-\left(d_P^{\ell}\right)^{q}\right)^{\mathfrak{x}}\right)\right)\right)^{\frac{1}{\mathfrak{x}}}},\sqrt[q]{1-\left(1-\left(\left(1-\left(1-\left(d_P^{u}\right)^{q}\right)^{\mathfrak{x}}\right)\right)\right)^{\frac{1}{\mathfrak{x}}}}\right]\end{array}\right)$$

$$=\left(\left[\left(\sqrt[q]{\left(m_P^{\ell}\right)^{q\mathfrak{x}}}\right)^{\frac{1}{\mathfrak{x}}},\left(\sqrt[q]{\left(m_P^{u}\right)^{q\mathfrak{x}}}\right)^{\frac{1}{\mathfrak{x}}}\right],\left[\sqrt[q]{\left(a_P^{\ell}\right)^{q}},\sqrt[q]{\left(a_P^{u}\right)^{q}}\right],\left[\sqrt[q]{\left(d_P^{\ell}\right)^{q}},\sqrt[q]{\left(d_P^{u}\right)^{q}}\right]\right)$$

$$=\left(\left[m_P^{\ell},m_P^{u}\right],\left[a_P^{\ell},a_P^{u}\right],\left[d_P^{\ell},d_P^{u}\right]\right)$$

Property 2: Let P_ϱ and $\acute{P}_\varrho$ be the collection of IVTSFNs: if $\left[m_\varrho^{\ell},m_\varrho^{u}\right]\geq\left[\acute{m}_\varrho{}^{\ell},\acute{m}_\varrho{}^{u}\right],\left[a_\varrho^{\ell},a_\varrho^{u}\right]\leq\left[\acute{a}_\varrho{}^{\ell},\acute{a}_\varrho{}^{u}\right],\left[d_\varrho^{\ell},d_\varrho^{u}\right]\leq\left[\acute{d}_\varrho{}^{\ell},\acute{d}_\varrho{}^{u}\right]$, for all ϱ then:

$$IVTSFMSM(P_1,P_2,\dots,P_n)\geq IVTSFMSM\left(\acute{P}_1,\acute{P}_2,\dots,\acute{P}_n\right)$$

Proof: By hypothesis, it is clear that $\mathfrak{x}\geq 1,\left[m_\varrho^{\ell},m_\varrho^{u}\right]\geq\left[\acute{m}_\varrho{}^{\ell},\acute{m}_\varrho{}^{u}\right]\geq 0,\left[a_\varrho^{\ell},a_\varrho^{u}\right]\leq\left[\acute{a}_\varrho{}^{\ell},\acute{a}_\varrho{}^{u}\right]\leq 0,\left[d_\varrho^{\ell},d_\varrho^{u}\right]\leq\left[\acute{d}_\varrho{}^{\ell},\acute{d}_\varrho{}^{u}\right]\leq 0$ and $\left[m_{i_\varrho}^{\ell},m_{i_\varrho}^{u}\right]\geq\left[\acute{m}_{i_\varrho}^{\ell},\acute{m}_{i_\varrho}^{u}\right]\geq 0,\left[a_{i_\varrho}^{\ell},a_{i_\varrho}^{u}\right]\leq\left[\acute{a}_{i_\varrho}^{\ell},\acute{a}_{i_\varrho}^{u}\right]\leq 0,\left[d_{i_\varrho}^{\ell},d_{i_\varrho}^{u}\right]\leq\left[\acute{d}_{i_\varrho}^{\ell},\acute{d}_{i_\varrho}^{u}\right]\leq 0$. Based on the given condition, for all j (j $=1,2,\dots,$ n; $\varrho=1,2,\dots,$ k) we obtain:

$$\left[\prod_{\varrho=1}^{\mathfrak{x}}m_{i\ \varrho}^{\ell},\prod_{\varrho=1}^{\mathfrak{x}}m_{i\ \varrho}^{u}\right]\geq\left[\prod_{\varrho=1}^{\mathfrak{x}}\acute{m}_{i\ \varrho}^{\ell},\prod_{\varrho=1}^{\mathfrak{x}}\acute{m}_{i\ \varrho}^{u}\right]\Rightarrow\left[1-\left(\prod_{\varrho=1}^{\mathfrak{x}}m_{i\ \varrho}^{\ell}\right)^{q},1-\left(\prod_{\varrho=1}^{\mathfrak{x}}m_{i\ \varrho}^{u}\right)^{q}\right]$$

$$\leq\left[1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\acute{m}_{i\ \varrho}^{\ell}\right)^{q},1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\acute{m}_{i\ \varrho}^{u}\right)^{q}\right]$$

$$\Rightarrow\left[\prod_{\substack{1\leq i_1\leq\cdots\\<i_{\mathfrak{x}}\leq n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}m_{i\ \varrho}^{\ell}\right)^{q}\right),\prod_{\substack{1\leq i_1\leq\cdots\\<i_{\mathfrak{x}}\leq n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}m_{i\ \varrho}^{u}\right)^{q}\right)\right]$$

$$\leq\left[\prod_{\substack{1\leq i_1\leq\cdots\\<i_{\mathfrak{x}}\leq n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\acute{m}_{i\ \varrho}^{\ell}\right)^{q}\right),\prod_{\substack{1\leq i_1\leq\cdots\\<i_{\mathfrak{x}}\leq n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\acute{m}_{i\ \varrho}^{u}\right)^{q}\right)\right]$$

$$\Rightarrow\left[\left(\sqrt[q]{1-\left(\prod_{\substack{1\leq i_1\leq\cdots\\<i_{\mathfrak{x}}\leq n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}m_{i\ \varrho}^{\ell}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}},\left(\sqrt[q]{1-\left(\prod_{\substack{1\leq i_1\leq\cdots\\<i_{\mathfrak{x}}\leq n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}m_{i\ \varrho}^{u}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}\right]$$

$$\geq\left[\left(\sqrt[q]{1-\left(\prod_{\substack{1\leq i_1\leq\cdots\\<i_{\mathfrak{x}}\leq n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\acute{m}_{i\ \varrho}^{\ell}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}},\left(\sqrt[q]{1-\left(\prod_{\substack{1\leq i_1\leq\cdots\\<i_{\mathfrak{x}}\leq n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\acute{m}_{i\ \varrho}^{u}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}\right]$$

Now, for abstinence

$$\left[a_{i_\varrho}^{\ell}, a_{i_\varrho}^{u}\right] \le \left[a_{i_\varrho}^{\prime\ell}, a_{i_\varrho}^{\prime u}\right] \Rightarrow \left[1-\left(a_{i_\varrho}^{\ell}\right)^{q}, 1-\left(a_{i_\varrho}^{u}\right)^{q}\right] \le \left[1-\left(a_{i_\varrho}^{\prime\ell}\right)^{q}, 1-\left(a_{i_\varrho}^{\prime u}\right)^{q}\right]$$

$$\Rightarrow \left[1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right), 1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)\right]$$

$$\le \left[1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\prime\ell}\right)^{q}\right), 1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\prime u}\right)^{q}\right)\right]$$

$$\left[\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}, \left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right]$$

$$\le \left[\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\prime\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}, \left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\prime u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right]$$

$$\left[\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}, \left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}\right]$$

$$\ge \left[\begin{array}{c}\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\prime\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right),\\ \left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\prime u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}},\end{array}\right]$$

$$\Rightarrow \left[\sqrt[q]{\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}}, \sqrt[q]{\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}}\right]$$

$$\ge \left[\sqrt[q]{\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\prime\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}}, \sqrt[q]{\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\prime u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}}\right]$$

As $\left[d_{i_\varrho}^{\ell}, d_{i_\varrho}^{u}\right] \le \left[d_{i_\varrho}^{\prime\ell}, d_{i_\varrho}^{\prime u}\right] \Rightarrow \left[1-\left(d_{i_\varrho}^{\ell}\right)^{q}, 1-\left(d_{i_\varrho}^{u}\right)^{q}\right] \ge \left[1-\left(d_{i_\varrho}^{\prime\ell}\right)^{q}, 1-\left(d_{i_\varrho}^{\prime u}\right)^{q}\right]$.

$$\Rightarrow \left[1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right), 1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)\right] \le \left[1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_{i_\varrho}^{\prime\ell}\right)^{q}\right), 1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_{i_\varrho}^{\prime u}\right)^{q}\right)\right]$$

$$\left[\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^x}},\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^x}}\right]$$

$$\le\left[\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(\acute{d}_{i_\varrho}^{\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^x}},\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(\acute{d}_{i_\varrho}^{u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^x}}\right]$$

$$\Rightarrow\left[\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}},\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}\right]$$

$$\ge\left[\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(\acute{d}_{i_\varrho}^{\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}},\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(\acute{d}_{i_\varrho}^{u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}\right]$$

$$\Rightarrow\left[\sqrt[q]{\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}},\sqrt[q]{\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}}\right]$$

$$\ge\left[\sqrt[q]{\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(\acute{d}_{i_\varrho}^{\ell}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}},\sqrt[q]{\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(\acute{d}_{i_\varrho}^{u}\right)^{q}\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}}\right]$$

From the above analysis, we obtain:

$$IVTSFMSM(P_1,P_2,\dots,P_n)\ge IVTSFMSM(\acute{P}_1,\acute{P}_2,\dots,\acute{P}_n)$$

Property 3: Let P_ϱ and $\acute{P}_\varrho$ be the collection of two IVTSFNs. Then:

$$IVTSFMSM(P_1,P_2,P_3,\dots P_n)=IVTSFMSM(\acute{P}_1,\acute{P}_2,\acute{P}_3,\dots,\acute{P}_n)$$

Proof: We know that $\acute{P}_\varrho$ is any permutation of P_ϱ then:

$$\mathrm{IV}TSFMSM(P_1,P_2,P_3,\dots,P_n)=\left(\frac{\bigoplus_{1\le i_1\le\cdots i_x\le n}\left(\otimes_{\varrho=1}^{x}P_{i_\varrho}\right)}{C_n^x}\right)^{\frac{1}{x}}$$

$$=\left(\frac{\bigoplus_{1\le i_1\le\cdots i_x\le n}\left(\otimes_{\varrho=1}^{x}\acute{P}_{i_\varrho}\right)}{C_n^x}\right)^{\frac{1}{x}}=IVTSFMSM(\acute{P}_1,\acute{P}_2,\acute{P}_3,\dots,\acute{P}_n)$$

Property 4: Let P_ϱ be the collection of IVTSFNs with:

$$P^-=\min P_\varrho=\left(\left[min\, m_\varrho^{\ell},min\, m_\varrho^{u}\right],\left[\max a_\varrho^{\ell},\max a_\varrho^{u}\right],\left[\max d_\varrho^{\ell},\max d_\varrho^{u}\right]\right)$$
$$P^+=\max P_\varrho=\left(\left[\max m_\varrho^{\ell},\max m_\varrho^{u}\right],\left[min\, a_\varrho^{\ell},min\, a_\varrho^{u}\right],\left[min\, d_\varrho^{\ell},min\, d_\varrho^{u}\right]\right)$$

Then

$$P^-\le\ IVTSFMSM(P_1,P_2,P_3\dots P_n)\le P^+$$

Proof: By using property 1 and property 2 we get:

$$IVTSFMSM(P_1, P_2, P_3, \dots, P_n) \geq IV\ TSFMSM(P^-, P^-, \dots, P^-) = P^-$$
$$IVTSFMSM(P_1, P_2, P_3, \dots, P_n) \leq IVTSFMSM(P^+, P^+, \dots, P^+) = P^+$$

By using the above information, we get:

$$P^- \leq IVTSFMSM(P_1, P_2, P_3, \dots, P_n) \leq P^+$$

Example 1: Let $P_1 = ([0.3,0.31],[0.5,0.51],[0.6,0.61]), P_2 = ([0.4,0.41],[0.6,0.61],[0.7,0.71]), P_3 = ([0.5,0.51],[0.4,0.41],[0.7,0.71]), P_4 = ([0.6,0.61],[0.5,0.51],[0.6,0.61])$ be four IVTSFNs. Now use the IVTSFMSM operator to aggregate these four IVTSFNs. Here we take $\mathfrak{x} = 2, q = 5$, then:

$$P_1 \otimes P_2 = \begin{pmatrix} [0.3 \times 0.4, 0.31 \times 0.41], \left[\sqrt[5]{0.5^5 + 0.6^5 - 0.5^5 \times 0.6^5}, \sqrt[5]{0.51^5 + 0.61^5 - 0.51^5 \times 0.61^5}\right], \\ \left[\sqrt[5]{0.6^5 + 0.7^5 - 0.6^5 \times 0.7^5}, \sqrt[5]{0.61^5 + 0.71^5 - 0.61^5 \times 0.71^5}\right] \end{pmatrix}$$
$$= ([0.12,0.13],[0.63,0.65],[0.74,0.76])$$

$$P_1 \otimes P_3 = \begin{pmatrix} [0.3 \times 0.5, 0.31 \times 0.51], \left[\sqrt[5]{0.5^5 + 0.4^5 - 0.5^5 \times 0.4^5}, \sqrt[5]{0.51^5 + 0.41^5 - 0.51^5 \times 0.41^5}\right], \\ \left[\sqrt[5]{0.6^5 + 0.7^5 - 0.6^5 \times 0.7^5}, \sqrt[5]{0.61^5 + 0.71^5 - 0.61^5 \times 0.71^5}\right] \end{pmatrix}$$
$$= ([0.15,0.16],[0.52,0.54],[0.74,0.76])$$

$$P_1 \otimes P_4 = \begin{pmatrix} [0.3 \times 0.6, 0.31 \times 0.61], \left[\sqrt[5]{0.5^5 + 0.5 - 0.5^5 \times 0.5^5}, \sqrt[5]{0.51^5 + 0.51 - 0.51^5 \times 0.51^5}\right], \\ \left[\sqrt[5]{0.6^5 + 0.6^5 - 0.6^5 \times 0.6^5}, \sqrt[5]{0.61^5 + 0.61^5 - 0.61^5 \times 0.61^5}\right] \end{pmatrix}$$
$$= ([0.18,0.19],[0.57,0.59],[0.68,0.70])$$

$$P_2 \otimes P_3 = \begin{pmatrix} [0.4 \times 0.5, 0.41 \times 0.51], \left[\sqrt[5]{0.6^5 + 0.4^5 - 0.6^5 \times 0.4^5}, \sqrt[5]{0.61^5 + 0.41^5 - 0.61^5 \times 0.41^5}\right], \\ \left[\sqrt[5]{0.7^5 + 0.7^5 - 0.7^5 \times 0.7^5}, \sqrt[5]{0.71^5 + 0.71^5 - 0.71^5 \times 0.71^5}\right] \end{pmatrix}$$
$$= ([0.20,0.21],[0.61,0.63],[0.79,0.81])$$

$$P_2 \otimes P_4 = \begin{pmatrix} [0.4 \times 0.6, 0.41 \times 0.61], \left[\sqrt[5]{0.6^5 + 0.5^5 - 0.6^5 \times 0.5^5}, \sqrt[5]{0.61^5 + 0.51^5 - 0.61^5 \times 0.51^5}\right], \\ \left[\sqrt[5]{0.7^5 + 0.6^5 - 0.7^5 \times 0.6^5}, \sqrt[5]{0.71^5 + 0.61^5 - 0.71^5 \times 0.61^5}\right] \end{pmatrix}$$
$$= ([0.24,0.25],[0.63,0.65],[0.74,0.76])$$

$$P_3 \otimes P_4 = \begin{pmatrix} [0.5 \times 0.6, 0.51 \times 0.61], \left[\sqrt[5]{0.4^5 + 0.5^5 - 0.4^5 \times 0.5^5}, \sqrt[5]{0.41^5 + 0.51^5 - 0.41^5 \times 0.51^5}\right], \\ \left[\sqrt[5]{0.7^5 + 0.6^5 - 0.7^5 \times 0.6^5}, \sqrt[5]{0.71^5 + 0.61^5 - 0.71^5 \times 0.61^5}\right] \end{pmatrix}$$
$$= ([0.30,0.31],[0.52,0.54],[0.74,0.76])$$

Using formula

$$IVTSFMSM(P_1, P_2, P_3, P_4) = \left(\frac{\oplus_{1 \leq i_1 \leq i_2 \leq 4}(P_{i_1} \otimes P_{i_2})}{C_4^2}\right)^{\frac{1}{2}} = ([0.47,0.49],[0.51,0.52],[0.54,0.56]).$$

Definition 7: Let $P_\varrho = (m_\varrho, \alpha_\varrho, d_\varrho)$ be a collection of IVTSFNs. Here $m_\varrho = [m_\varrho^\ell, m_\varrho^u], a_\varrho = [a_\varrho^\ell, a_\varrho^u]$ and $d_\varrho = [d_\varrho^\ell, d_\varrho^u]$ Then the IVTSFWMSM operator is elaborated by:

$$IVTSFWMSM(P_1, P_2, \dots, P_n) = \left(\frac{\oplus_{1 \leq i_1 \leq \cdots i_\mathfrak{x} \leq n}\left(\otimes_{\varrho=1}^{\mathfrak{x}} P_{i_\varrho}\right)^{\omega}}{C_n^{\mathfrak{x}}}\right)^{\frac{1}{\mathfrak{x}}}$$

Where binomial coefficient is denoted by $\mathrm{C}_\mathrm{n}^\mathrm{k}$ and $(i_1, i_2, \dots, i_\mathfrak{x})$ represent the k-tuple combination of $(1,2, \dots, n)$ and $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of P_ϱ and $\omega_\varrho > 0, \sum_{\varrho=1}^{n} \omega_\varrho = 1$.

Theorem 2: Let $P_\varrho = (m_\varrho, a_\varrho, d_\varrho)$ be the collection of IVTSFNs. Here $m_\varrho = [m_\varrho^\ell, m_\varrho^u], a_\varrho = [a_\varrho^\ell, a_\varrho^u]$ and $d_\varrho = [d_\varrho^\ell, d_\varrho^u]$. Then by using the idea of IVTSFWMSM operators we obtain:

$IVTSFWMSM(P_1, P_2, \ldots, P_n) =$

$$\left(\begin{array}{c}\left[\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x}\left(m_{i_\varrho}^{\ell}\right)^{w_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^x}}}\right)^{\frac{1}{x}},\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x}\left(m_{i_\varrho}^{u}\right)^{w_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^x}}}\right)^{\frac{1}{x}}\right],\\ \left[\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}},\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}}\right],\\ \left[\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}},\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\prod_{\varrho=1}^{x}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}}\right]\end{array}\right)$$

Where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the weight vector of P_ϱ i.e., with a rule that is $\omega_\varrho > 0, \sum_{\varrho=1}^{n} \omega_\varrho = 1$.

Proof: By using Definition (7) we obtain:

$$\bigotimes_{\varrho=1}^{x}\left(P_{i_\varrho}\right)^{w_{i_\varrho}} = \left(\left[\prod_{\varrho=1}^{x}\left(m_{i_\varrho}^{\ell}\right)^{w_{i_\varrho}},\prod_{\varrho=1}^{x}\left(m_{i_\varrho}^{u}\right)^{w_{i_\varrho}}\right],\left[\prod_{\varrho=1}^{x}\left(a_{i_\varrho}^{\ell}\right)^{w_{i_\varrho}},\prod_{\varrho=1}^{x}\left(a_{i_\varrho}^{u}\right)^{w_{i_\varrho}}\right],\left[\sqrt[q]{1-\prod_{\varrho=1}^{x}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}},\sqrt[q]{1-\prod_{\varrho=1}^{x}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}}\right]\right)$$

$$\bigoplus_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(\bigotimes_{\varrho=1}^{x}\left(P_{i_\varrho}\right)^{w_{i_\varrho}}\right) = \left(\begin{array}{c}\left[\sqrt[q]{1-\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x}\left(m_{i_\varrho}^{\ell}\right)^{w_{i_\varrho}}\right)^{q}\right)},\sqrt[q]{1-\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x}\left(m_{i_\varrho}^{u}\right)^{w_{i_\varrho}}\right)^{q}\right)}\right],\\ \left[\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\sqrt[q]{1-\prod_{\varrho=1}^{x}\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}},\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\sqrt[q]{1-\prod_{\varrho=1}^{x}\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}}\right],\\ \left[\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\sqrt[q]{1-\prod_{\varrho=1}^{x}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}},\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\sqrt[q]{1-\prod_{\varrho=1}^{x}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}}\right]\end{array}\right)$$

$$\frac{1}{C_n^{\mathfrak{x}}}\bigoplus_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(\bigotimes_{\varrho=1}^{\mathfrak{x}}P_{i_\varrho}\right)$$

$$=\left(\begin{array}{l}\left[\sqrt[q]{1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod\limits_{\varrho=1}^{\mathfrak{x}}\left(m_{i_\varrho}^{\ell}\right)^{w_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}},\sqrt[q]{1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod\limits_{\varrho=1}^{\mathfrak{x}}\left(m_{i_\varrho}^{u}\right)^{w_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right],\\ \left[\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\sqrt[q]{1-\prod\limits_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}},\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\sqrt[q]{1-\prod\limits_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right],\\ \left[\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\sqrt[q]{1-\prod\limits_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}},\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\sqrt[q]{1-\prod\limits_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right]\end{array}\right)$$

$$IVTSFWMSM_{\omega}(P_1,P_2,\ldots,P_n)$$

$$=\left(\begin{array}{l}\left[\sqrt[q]{1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod\limits_{\varrho=1}^{\mathfrak{x}}\left(m_{i_\varrho}^{\ell}\right)^{w_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}},\sqrt[q]{1-\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod\limits_{\varrho=1}^{\mathfrak{x}}\left(m_{i_\varrho}^{u}\right)^{w_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right],\\ \left[\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\sqrt[q]{1-\prod\limits_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}},\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\sqrt[q]{1-\prod\limits_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right],\\ \left[\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\sqrt[q]{1-\prod\limits_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}},\left(\prod\limits_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\sqrt[q]{1-\prod\limits_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right]\end{array}\right)$$

$$IVTSFWMSM_{\omega}(P_1,P_2,\dots,P_n)$$

$$= \left(\begin{array}{c} \left[\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\left(m_{i_\varrho}^{\ell}\right)^{w_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}},\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\left(m_{i_\varrho}^{u}\right)^{w_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}\right], \\ \left[\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}},\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}}\right] \\ ,\left[\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}},\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}}\right] \end{array} \right)$$

Some Special Cases for TSFWMSM

For $\mathfrak{x}=1$ the IVTSFWMSM operator is reduced into IVTSF weighted averaging (IVTSFWA) operator.

$$IVTSFWMSM_{\omega}(P_1,P_2,\dots,P_n)$$

$$= \left(\begin{array}{c} \left[\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{1}\left(m_{i_\varrho}^{\ell}\right)^{w_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{1}}}}\right)^{\frac{1}{1}},\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{1}\left(m_{i_\varrho}^{u}\right)^{w_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{1}}}}\right)^{\frac{1}{1}}\right], \\ \left[\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{1}\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^{1}}}\right)^{\frac{1}{1}}},\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{1}\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^{1}}}\right)^{\frac{1}{1}}}\right], \\ \left[\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{1}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^{1}}}\right)^{\frac{1}{1}}},\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{1}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^{1}}}\right)^{\frac{1}{1}}}\right] \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\sqrt[q]{1-\left(\prod_{1\le i_1\le n}\left(1-\left(m_{i_1}^{\ell}\right)^{q\omega_{i_1}}\right)\right)^{\frac{1}{n}}}, \sqrt[q]{1-\left(\prod_{1\le i_1\le n}\left(1-\left(m_{i_1}^{u}\right)^{q\omega_{i_1}}\right)\right)^{\frac{1}{n}}} \right], \\ \left[\left(\prod_{1\le i_1\le n}\sqrt[q]{1-\prod_{\varrho=1}^{1}\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{n}}, \left(\prod_{1\le i_1\le n}\sqrt[q]{1-\prod_{\varrho=1}^{1}\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{n}} \right], \\ \left[\left(\prod_{1\le i_1\le n}\sqrt[q]{1-\prod_{\varrho=1}^{1}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{n}}, \left(\prod_{1\le i_1\le n}\sqrt[q]{1-\prod_{\varrho=1}^{1}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{n}} \right] \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\sqrt[q]{1-\left(\prod_{1\le i_1\le n}\left(1-\left(m_{i_1}^{\ell}\right)^{q\omega_{i_1}}\right)\right)^{\frac{1}{n}}}, \sqrt[q]{1-\left(\prod_{1\le i_1\le n}\left(1-\left(m_{i_1}^{u}\right)^{q\omega_{i_1}}\right)\right)^{\frac{1}{n}}} \right], \\ \left[\left(\prod_{1\le i_1\le n}\sqrt[q]{1-\left(1-\left(a_{i_1}^{\ell}\right)^{q}\right)^{\omega_{i_1}}}\right)^{\frac{1}{n}}, \left(\prod_{1\le i_1\le n}\sqrt[q]{1-\left(1-\left(a_{i_1}^{u}\right)^{q}\right)^{\omega_{i_1}}}\right)^{\frac{1}{n}} \right], \\ \left[\left(\prod_{1\le i_1\le n}\sqrt[q]{1-\left(1-\left(d_{i_1}^{\ell}\right)^{q}\right)^{\omega_{i_1}}}\right)^{\frac{1}{n}}, \left(\prod_{1\le i_1\le n}\sqrt[q]{1-\left(1-\left(d_{i_1}^{u}\right)^{q}\right)^{\omega_{i_1}}}\right)^{\frac{1}{n}} \right] \end{array} \right)$$

Since we take $i_1 = \varrho$ then

$$= \left(\begin{array}{c} \left[\sqrt[q]{1-\left(\prod_{\varrho=1}^{n}\left(1-\left(m_{\varrho}^{\ell}\right)^{q\omega_{\varrho}}\right)\right)^{\frac{1}{n}}}, \sqrt[q]{1-\left(\prod_{\varrho=1}^{n}\left(1-\left(m_{\varrho}^{u}\right)^{q\omega_{\varrho}}\right)\right)^{\frac{1}{n}}} \right], \\ \left[\left(\prod_{\varrho=1}^{n}\sqrt[q]{1-\left(1-\left(a_{\varrho}^{\ell}\right)^{q}\right)^{\omega_{\varrho}}}\right)^{\frac{1}{n}}, \left(\prod_{\varrho=1}^{n}\sqrt[q]{1-\left(1-\left(a_{\varrho}^{u}\right)^{q}\right)^{\omega_{\varrho}}}\right)^{\frac{1}{n}} \right], \\ \left[\left(\prod_{\varrho=1}^{n}\sqrt[q]{1-\left(1-\left(d_{\varrho}^{\ell}\right)^{q}\right)^{\omega_{\varrho}}}\right)^{\frac{1}{n}}, \left(\prod_{\varrho=1}^{n}\sqrt[q]{1-\left(1-\left(d_{\varrho}^{u}\right)^{q}\right)^{\omega_{\varrho}}}\right)^{\frac{1}{n}} \right] \end{array} \right)$$

For $\mathfrak{x} = 2$ the IVTSFWMSM operator is reduced into IVTSF weighted Bonferroni mean (IVTSFWBM) operator:

$IVTSFWMSM^{2}(P_1, P_2, \dots, P_n)$

$$= \left(\begin{array}{c} \left[\sqrt[q]{1-\left(\prod_{1\le i_1\le i_2\le n}\left(1-\left(\prod_{\varrho=1}^{2}\left(m_{i_\varrho}^{\ell}\right)^{w_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^2}}}, \sqrt[q]{1-\left(\prod_{1\le i_1\le i_2\le n}\left(1-\left(\prod_{\varrho=1}^{2}\left(m_{i_\varrho}^{u}\right)^{w_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^2}}} \right], \\ \left[\left(\prod_{1\le i_1\le i_2\le n}\sqrt[q]{1-\prod_{\varrho=1}^{2}\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{C_n^2}}, \left(\prod_{1\le i_1\le i_2\le n}\sqrt[q]{1-\prod_{\varrho=1}^{2}\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{C_n^2}} \right], \\ \left[\left(\prod_{1\le i_1\le i_2\le n}\sqrt[q]{1-\prod_{\varrho=1}^{2}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{C_n^2}}, \left(\prod_{1\le i_1\le i_2\le n}\sqrt[q]{1-\prod_{\varrho=1}^{2}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}}\right)^{\frac{1}{C_n^2}} \right] \end{array} \right)$$

$IVTSFWMSM^{2}(P_1, P_2, \dots, P_n)$

$$= \left(\begin{array}{c} \left[\left(\sqrt[q]{1-\left(\prod_{1\le i_1\le i_2\le n}\left(1-\left(\left(m_{i_1}^{\ell}\right)^{\omega_{i_1}}\left(m_{i_2}^{\ell}\right)^{\omega_{i_1}}\right)^{q}\right)\right)^{\frac{2}{n(n-1)}}} \right), \left(\sqrt[q]{1-\left(\prod_{1\le i_1\le i_2\le n}\left(1-\left(\left(m_{i_1}^{u}\right)^{\omega_{i_1}}\left(m_{i_2}^{u}\right)^{\omega_{i_1}}\right)^{q}\right)\right)^{\frac{2}{n(n-1)}}} \right) \right], \\ \left[\left(\prod_{1\le i_1\le i_2\le n}\sqrt[q]{1-\left(1-\left(a_{i_1}^{\ell}\right)^{q}\right)^{\omega_{i_1}}\left(1-\left(a_{i_2}^{\ell}\right)^{q}\right)^{\omega_{i_2}}}\right)^{\frac{2}{n(n-1)}}, \left(\prod_{1\le i_1\le i_2\le n}\sqrt[q]{1-\left(1-\left(a_{i_1}^{u}\right)^{q}\right)^{\omega_{i_1}}\left(1-\left(a_{i_2}^{u}\right)^{q}\right)^{\omega_{i_2}}}\right)^{\frac{2}{n(n-1)}} \right], \\ \left[\left(\prod_{1\le i_1\le i_2\le n}\sqrt[q]{1-\left(1-\left(d_{i_1}^{\ell}\right)^{q}\right)^{\omega_{i_1}}\left(1-\left(d_{i_2}^{\ell}\right)^{q}\right)^{\omega_{i_2}}}\right)^{\frac{2}{n(n-1)}}, \left(\prod_{1\le i_1\le i_2\le n}\sqrt[q]{1-\left(1-\left(d_{i_1}^{u}\right)^{q}\right)^{\omega_{i_1}}\left(1-\left(d_{i_2}^{u}\right)^{q}\right)^{\omega_{i_2}}}\right)^{\frac{2}{n(n-1)}} \right] \end{array} \right)$$

$= (IVTSFWBM)^{(1,1)}(P_1, P_2, \dots, P_n)$

For $\mathfrak{x} = n$ the IVTSFMSM operator is reduced into an IVTSFWBM operator if $(p = 1, q = 0)$

$$IVTSFWMSM_{\omega}^{n}(P_1, P_2, \dots, P_n) = \left(\frac{\bigoplus_{1\le i_1\le \cdots i_{\mathfrak{x}}\le n}\left(\otimes_{\varrho=1}^{\mathfrak{x}} P_{i_\varrho}\right)^{\omega_{i_\varrho}}}{C_n^n} \right)^{\frac{1}{n}}$$

$IVTSFWMSM_{\omega}^{n}(P_1, P_2, \dots, P_n)$

$$= \left(\begin{array}{c} \left[\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{n}\left(m_{i_\varrho}^{\ell}\right)^{w_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^n}}} \right)^{\frac{1}{n}}, \left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{n}\left(m_{i_\varrho}^{u}\right)^{w_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^n}}} \right)^{\frac{1}{n}} \right], \\ \left[\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{n}\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^n}}\right)^{\frac{1}{n}}}, \sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{n}\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^n}}\right)^{\frac{1}{n}}} \right], \\ \left[\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{n}\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^n}}\right)^{\frac{1}{n}}}, \sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{n}\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)^{w_{i_\varrho}}\right)\right)^{\frac{1}{C_n^n}}\right)^{\frac{1}{n}}} \right] \end{array} \right)$$

$$IVTSFWMSM_{\omega}^{n}(P_1,P_2,\ldots,P_n)=\begin{pmatrix}\left[\left(\prod_{\varrho=1}^{n}\left(m_{i_\varrho}^{\ell}\right)^{\omega_{i_\varrho}}\right)^{\frac{1}{n}},\left(\prod_{\varrho=1}^{n}\left(m_{i_\varrho}^{u}\right)^{\omega_{i_\varrho}}\right)^{\frac{1}{n}}\right],\\ \left[\sqrt[q]{1-\prod_{\varrho=1}^{n}\left(\left(1-\left(a_{i_\varrho}^{\ell}\right)^{q}\right)^{\omega_{i_\varrho}}\right)^{\frac{1}{n}}},\sqrt[q]{1-\prod_{\varrho=1}^{n}\left(\left(1-\left(a_{i_\varrho}^{u}\right)^{q}\right)^{\omega_{i_\varrho}}\right)^{\frac{1}{n}}},\right]\\ \left[\sqrt[q]{1-\prod_{\varrho=1}^{n}\left(\left(1-\left(d_{i_\varrho}^{\ell}\right)^{q}\right)^{\omega_{i_\varrho}}\right)^{\frac{1}{n}}},\sqrt[q]{1-\prod_{\varrho=1}^{n}\left(\left(1-\left(d_{i_\varrho}^{u}\right)^{q}\right)^{\omega_{i_\varrho}}\right)^{\frac{1}{n}}}\right]\end{pmatrix}$$

If we take $i_\varrho=\varrho$ then

$$IVTSFWMSM_{\omega}^{n}(P_1,P_2,\ldots,P_n)=\begin{pmatrix}\left[\left(\prod_{\varrho=1}^{n}\left(m_{\varrho}^{\ell}\right)^{\omega_{\varrho}}\right)^{\frac{1}{n}},\left(\prod_{\varrho=1}^{n}\left(m_{\varrho}^{u}\right)^{\omega_{\varrho}}\right)^{\frac{1}{n}}\right],\\ \left[\sqrt[q]{1-\left(\prod_{\varrho=1}^{n}\left(\left(1-\left(a_{\varrho}^{\ell}\right)^{q}\right)^{\omega_{\varrho}}\right)\right)^{\frac{1}{n}}},\sqrt[q]{1-\left(\prod_{\varrho=1}^{n}\left(\left(1-\left(a_{\varrho}^{u}\right)^{q}\right)^{\omega_{\varrho}}\right)\right)^{\frac{1}{n}}},\right]\\ \left[\sqrt[q]{1-\left(\prod_{\varrho=1}^{n}\left(\left(1-\left(d_{\varrho}^{\ell}\right)^{q}\right)^{\omega_{\varrho}}\right)\right)^{\frac{1}{n}}},\sqrt[q]{1-\left(\prod_{\varrho=1}^{n}\left(\left(1-\left(d_{\varrho}^{u}\right)^{q}\right)^{\omega_{\varrho}}\right)\right)^{\frac{1}{n}}}\right]\end{pmatrix}$$

$$=IVTSFWBM^{(1,0)}(P_1,P_2,\ldots,P_n)$$

Example: 2 Let $P_1=([0.3,0.31],[0.5,0.51],[0.6,0.61]),P_2=([0.4,0.41],[0.6,0.61],[0.7,0.71]),P_3=([0.5,0.51],[0.4,0.41],[0.7,0.71]),P_4=([0.6,0.61],[0.5,0.51],[0.6,0.61])$ be four IVTSFNs. Now use the IVTSFWMSM operator to aggregate these four IVTSFNs. Here we take $\mathfrak{x}=2,q=3$ that fulfill our requirements and $\omega=(0.3,0.1,0.4,0.2)$ be the weight vector of four IVTSFNs.

$(P_1)^{0.3}\otimes(P_2)^{0.1}$

$$=\begin{pmatrix}[0.6^{0.3}\times0.6^{0.1},0.61^{0.3}\times0.61^{0.1}],\left[\sqrt[3]{1-(1-0.5^3)^{0.3}\times(1-0.6^3)^{0.1}},\sqrt[3]{1-(1-0.51^3)^{0.3}\times(1-0.61^3)^{0.1}}\right],\\ \left[\sqrt[3]{1-(1-0.8^3)^{0.3}\times(1-0.2^3)^{0.1}},\sqrt[3]{1-(1-0.81^3)^{0.3}\times(1-0.21^3)^{0.1}}\right]\end{pmatrix}$$

$=([0.81,0.82],[0.39,0.40],[0.57,0.59])$

$(P_1)^{0.3}\otimes(P_3)^{0.4}$

$$=\begin{pmatrix}[0.6^{0.3}\times0.4^{0.4},0.61^{0.3}\times0.41^{0.4}],\left[\sqrt[3]{1-(1-0.5^3)^{0.3}\times(1-0.4^3)^{0.4}},\sqrt[3]{1-(1-0.51^3)^{0.3}\times(1-0.41^3)^{0.4}}\right],\\ \left[\sqrt[3]{1-(1-0.8^3)^{0.3}\times(1-0.3^3)^{0.4}},\sqrt[3]{1-(1-0.81^3)^{0.3}\times(1-0.31^3)^{0.4}}\right]\end{pmatrix}$$

$=([0.71,0.72],[0.40,0.41],[0.58,0.60])$

$(P_1)^{0.3}\otimes(P_4)^{.0.2}$

$$=\begin{pmatrix}[0.6^{0.3}\times0.5^{0.2},0.61^{0.3}\times0.51^{0.2}],\left[\sqrt[3]{1-(1-0.5^3)^{0.3}\times(1-0.7^3)^{0.2}},\sqrt[3]{1-(1-0.51^3)^{0.3}\times(1-0.71^3)^{0.2}}\right],\\ \left[\sqrt[3]{1-(1-0.8^3)^{0.3}\times(1-0.4^3)^{0.2}},\sqrt[3]{1-(1-0.81^3)^{0.3}\times(1-0.41^3)^{0.2}}\right]\end{pmatrix}$$

$=([0.74,0.75],[0.48,0.50],[0.58,0.60])$

$(P_2)^{0.1}\otimes(P_3)^{0.4}$

$$=\begin{pmatrix}[0.6^{0.1}\times0.4^{0.4},0.61^{0.1}\times0.41^{0.4}],\left[\sqrt[3]{1-(1-0.6^3)^{0.1}\times(1-0.4^3)^{0.4}},\sqrt[3]{1-(1-0.61^3)^{0.1}\times(1-0.41^3)^{0.4}}\right],\\ \left[\sqrt[3]{1-(1-0.2^3)^{0.1}\times(1-0.3^3)^{0.4}},\sqrt[3]{1-(1-0.21^3)^{0.1}\times(1-0.31^3)^{0.4}}\right]\end{pmatrix}$$

$=([0.65,0.66],[0.36,0.38],[0.22,0.24])$

$(P_2)^{0.1}\otimes(P_4)^{0.2}$

$$=\begin{pmatrix}[0.6^{0.1}\times0.5^{0.2},0.61^{0.1}\times0.51^{0.2}],\left[\sqrt[3]{1-(1-0.6^3)^{0.1}\times(1-0.7^3)^{0.2}},\sqrt[3]{1-(1-0.61^3)^{0.1}\times(1-0.71^3)^{0.2}}\right],\\ \left[\sqrt[3]{1-(1-0.2^3)^{0.1}\times(1-0.4^3)^{0.2}},\sqrt[3]{1-(1-0.21^3)^{0.1}\times(1-0.41^3)^{0.2}}\right]\end{pmatrix}$$

$=([0.82,0.83],[0.46,0.48],[0.24,0.26])$

$$(P_3)^{0.4} \otimes (P_4)^{0.2}$$
$$= \begin{pmatrix} [0.4^{0.4} \times 0.5^{0.2}, 0.41^{0.4} \times 0.51^{0.2}], \left[\sqrt[3]{1-(1-0.4^3)^{0.4} \times (1-0.7^3)^{0.2}}, \sqrt[3]{1-(1-0.41^3)^{0.4} \times (1-0.71^3)^{0.2}}\right], \\ \left[\sqrt[3]{1-(1-0.3^3)^{0.4} \times (1-0.4^3)^{0.2}}, \sqrt[3]{1-(1-0.31^3)^{0.4} \times (1-0.41^3)^{0.2}}\right] \end{pmatrix}$$
$$= ([0.60,0.61],[0.47,0.48],[0.28,0.30])$$

Using the formula

$$TSFWMSM_{\omega}^{2}(P_1,P_2,\dots,P_n) = \left(\frac{\bigoplus_{1\le i_1\le\cdots i_{\mathfrak{x}}\le n}\left(\otimes_{\varrho=1}^{\mathfrak{x}} P_{i_\varrho}\right)^{\omega}}{C_n^{\mathfrak{x}}}\right)^{\frac{1}{\mathfrak{x}}} = \left(\frac{\bigoplus_{1\le i_1\le\cdots i_{\mathfrak{x}}\le n}\left(\otimes_{\varrho=1}^{\mathfrak{x}} P_{i_\varrho}\right)^{\omega}}{C_4^{2}}\right)^{\frac{1}{2}}$$
$$= ([0.35,0.36],[0.033,0.040],[0.42,0.43])$$

4. IVTSFDMSM and IVTSFWDMSM Operators

In this study, we combine the idea of DMSM and WDMSM operators with TSFSs to investigate the idea of IVTSFDMSM and IVTSFWDMSM operators and discussed their different properties.

Definition 8: Let $P_\varrho = (m_\varrho, a_\varrho, d_\varrho)$. Here $m_\varrho = (m_\varrho^\ell, m_\varrho^u)$, $a_\varrho = (a_\varrho^\ell, a_\varrho^u)$ and $d_\varrho = (d_\varrho^\ell, d_\varrho^u)$ be the collection of IVTSFNs. Then the IVTSFDMSM is elaborated by:

$$IVTSFDMSM^{K}\left((P_1,P_2,\dots,P_n)\right) = \frac{1}{\mathfrak{x}}\left(\bigoplus_{\substack{1\le i_1\le\cdots\\ <i_{\mathfrak{x}}\le n}}\left(\bigotimes_{\varrho=1}^{\mathfrak{x}} P_{i_\varrho}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)$$

Where binomial coefficient is denoted by $C_n^{\mathfrak{x}}$ with $(i_1, i_2, \dots, i_{\mathfrak{x}})$ all the k-tuple combinations of $(1,2,\dots,n)$.

Theorem 3: Let $P_\varrho = (m_\varrho, a_\varrho, d_\varrho)$ be the collection of IVTSFNs. Here $m_\varrho = (m_\varrho^\ell, m_\varrho^u)$, $a_\varrho = (a_\varrho^\ell, a_\varrho^u)$ and $d_\varrho = (d_\varrho^\ell, d_\varrho^u)$. Then by using the idea of IVTSFDMSM operators we obtain:

$$IVTSFDMSM(P_1,P_2,\dots,P_n)$$
$$= \left(\begin{array}{c} \left[\sqrt[q]{1-\left(1-\prod_{\substack{1\le i_1\le\cdots\\ <i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(m_{i\,\varrho}^{\ell}\right)^q\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}}, \sqrt[q]{1-\left(1-\prod_{\substack{1\le i_1\le\dots\\ <i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(m_{i\,\varrho}^{u}\right)^q\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}}\right], \\ \left[\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\ <i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} a_{i\,\varrho}^{\ell}\right)^q\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}, \left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\ <i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} a_{i\,\varrho}^{u}\right)^q\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}\right], \\ \left[\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\ <i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} d_{i\,\varrho}^{\ell}\right)^q\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}, \left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\ <i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} d_{i\,\varrho}^{u}\right)^q\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}\right] \end{array}\right)$$

Proof: By using Definition (8) we obtain:

$$\bigotimes_{\varrho=1}^{\mathfrak{x}} P_{i_\varrho} = \left(\begin{array}{c} \left[\sqrt[q]{1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(m_{i\,\varrho}^{\ell}\right)^q\right)}, \sqrt[q]{1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(m_{i\,\varrho}^{u}\right)^q\right)}\right], \\ \left[\prod_{\varrho=1}^{\mathfrak{x}} a_{i\,\varrho}^{\ell}, \prod_{\varrho=1}^{\mathfrak{x}} a_{i\,\varrho}^{u}\right], \left[\prod_{\varrho=1}^{\mathfrak{x}} d_{i\,\varrho}^{\ell}, \prod_{\varrho=1}^{\mathfrak{x}} d_{i\,\varrho}^{u}\right] \end{array}\right)$$

$$\left(\bigotimes_{\varrho=1}^{\mathfrak{x}} P_{i_\varrho}\right)^{\frac{1}{C_n^{\mathfrak{x}}}} = \left(\begin{array}{c}\left[\left(\sqrt[q]{1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(m_{i_\varrho}^{\ell}\right)^q\right)}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}, \left(\sqrt[q]{1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(m_{i_\varrho}^{\ell}\right)^q\right)}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right], \\ \left[\sqrt[q]{1-\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} a_{i_\varrho}^{\ell}\right)^q\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}, \sqrt[q]{1-\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} a_{i_\varrho}^{u}\right)^q\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right], \\ \left[\sqrt[q]{1-\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} d_{i_\varrho}^{\ell}\right)^q\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}, \sqrt[q]{1-\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} d_{i_\varrho}^{u}\right)^q\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right]\end{array}\right)$$

$$\bigotimes_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(\bigotimes_{\varrho=1}^{\mathfrak{x}} P_{i_\varrho}\right)^{\frac{1}{C_n^{\mathfrak{x}}}} = \left(\begin{array}{c}\left[\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(\sqrt[q]{1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(m_{i_\varrho}^{\ell}\right)^q\right)}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}, \prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(\sqrt[q]{1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(m_{i_\varrho}^{u}\right)^q\right)}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right], \\ \left[\sqrt[q]{1-\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} a_{i_\varrho}^{\ell}\right)^q\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}, \sqrt[q]{1-\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} a_{i_\varrho}^{u}\right)^q\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right] \\ ,\left[\sqrt[q]{1-\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} d_{i_\varrho}^{\ell}\right)^q\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}, \sqrt[q]{1-\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} d_{i_\varrho}^{u}\right)^q\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right]\end{array}\right)$$

$$IVTSFDMSM(P_1, P_2, \dots, P_n)$$

$$= \left(\begin{array}{c}\left[\sqrt[q]{1-\left(1-\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(m_{i_\varrho}^{\ell}\right)^q\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}}, \sqrt[q]{1-\left(1-\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(m_{i_\varrho}^{u}\right)^q\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}}\right], \\ \left[\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} a_{i_\varrho}^{\ell}\right)^q\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}, \left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} a_{i_\varrho}^{u}\right)^q\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}\right], \\ \left[\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} d_{i_\varrho}^{\ell}\right)^q\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}, \left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}} d_{i_\varrho}^{u}\right)^q\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}\right]\end{array}\right)$$

Moreover, the ideas of idempotency, monotonicity, commutations, and boundedness are developed.

Property 5: Let $P_\varrho = (m_\varrho, a_\varrho, d_\varrho)$ be the collection of IVTSFNs. Here $m_\varrho = (m_\varrho^\ell, m_\varrho^u)$, $a_\varrho = (a_\varrho^\ell, a_\varrho^u)$ and $d_\varrho = (d_\varrho^\ell, d_\varrho^u)$. If $P_\varrho = P$ then:

$IVTSFDMSM^{\mathfrak{x}}(P_1, P_2, P_3, \dots, P_n) = P$.

Property 6: Let P_ϱ and $\acute{P}_\varrho$ be the collection of IVTSFNs: if $[m_\varrho^\ell, m_\varrho^u] \ge [\acute{m}_\varrho{}^\ell, \acute{m}_\varrho{}^u]$, $[a_\varrho^\ell, a_\varrho^u] \le [\acute{a}_\varrho{}^\ell, \acute{a}_\varrho{}^u]$, $[d_\varrho^\ell, d_\varrho^u] \le [\acute{d}_\varrho{}^\ell, \acute{d}_\varrho{}^u]$, for all ϱ. Then:

$IVTSFDMSM^{\mathfrak{x}}(P_1, P_2, \dots, P_n) \ge IVTSFDMSM^{\mathfrak{x}}(\acute{P}_1, \acute{P}_2, \dots, \acute{P}_n)$.

Property 7: Let P_ϱ and $\acute{P}_\varrho$ be the collection of two IVTSFNs. Then:

$IVTSFDMSM^{\mathfrak{x}}(P_1, P_2, \dots, P_n) = IVTSFDMSM^{\mathfrak{x}}(\acute{P}_1, \acute{P}_2, \dots, \acute{P}_n)$.

Property 8: Let P_ϱ be the collection of IVTSFNs with

$$P^- = \min P_\varrho = \left(\left[min\, m_\varrho^\ell, min\, m_\varrho^u\right], \left[\max a_\varrho^\ell, \max a_\varrho^u\right], \left[\max d_\varrho^\ell, \max d_\varrho^u\right]\right)$$

$$P^- = \max P_\varrho = \left(\left[\max m_\varrho^\ell, \max m_\varrho^u\right], \left[min\, a_\varrho^\ell, min\, a_\varrho^u\right], \left[min\, d_\varrho^\ell, min\, d_\varrho^u\right]\right)$$

Then

$$P^- \le IVTSFDMSM(P_1, P_2, P_3 \dots P_n) \le P^+$$

Proof: By using property 1 and property 2, we get:

$$IVTSFDMSM^{(\mathfrak{x})}(P_1, P_2, P_3, \dots, P_n) \ge IVTSFDMSM^{(\mathfrak{x})}(P^-, P^-, \dots, P^-) = P^-$$

$$IVTSFDMSM^{(\mathfrak{x})}(P_1, P_2, P_3, \dots, P_n) \le IVTSFDMSM^{(\mathfrak{x})}(P^+, P^+, \dots, P^+) = P^+$$

Definition 9: Let $P_\varrho = (m_\varrho, a_\varrho, d_\varrho)$ be the collection of IVTSFNs. Here $m_\varrho = (m_\varrho^\ell, m_\varrho^u), a_\varrho = (a_\varrho^\ell, a_\varrho^u)$ and $d_\varrho = (d_\varrho^\ell, d_\varrho^u)$. Then the IVTSFWDMSM operator is elaborated by:

$$IVTSFWDMSM(P_1, P_2, \dots, P_n) = \frac{1}{\mathfrak{x}}\left(\bigoplus_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(\bigotimes_{\varrho=1}^{\mathfrak{x}} \left(\omega_{i_\varrho} \otimes P_{i_\varrho}\right)\right)^{\frac{1}{C_n^\mathfrak{x}}}\right)$$

Where C_n^k represented by a binomial coefficient and $(i_1, i_2, \dots, i_\mathfrak{x})$ represent the k-tuple combination of $(1,2, \dots, n)$ and $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of P_ϱ and $\omega_\varrho > 0, \sum_{\varrho=1}^n \omega_\varrho = 1$.

Theorem 4: Let $p_\varrho = (m_\varrho, a_\varrho, d_\varrho)$ be the collection of IVTSFNs. Here $m_\varrho = (m_\varrho^\ell, m_\varrho^u), a_\varrho = (a_\varrho^\ell, a_\varrho^u)$ and $d_\varrho = (d_\varrho^\ell, d_\varrho^u)$. Then by using the idea of IVTSFWDMSM operators, we obtain:

$$IVTSFWDMSM^{\mathfrak{x}}(P_1, P_2, P_3, \dots, P_n) = \left(\frac{\bigoplus_{1 \le i_1 \le \cdots i_\mathfrak{x} \le n} \left(\bigotimes_{\varrho=1}^{\mathfrak{x}} \left(P_{i_\varrho}\right)^{\omega_{i_\varrho}}\right)}{C_n^\mathfrak{x}}\right)^{\frac{1}{\mathfrak{x}}}$$

$$= \left(\begin{array}{c}
\left[\sqrt[q]{1 - \left(1 - \prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(m_{i_\varrho}^\ell\right)^q\right)^{w_{i_\varrho}}\right)^{\frac{1}{C_n^\mathfrak{x}}}\right)^{\frac{1}{\mathfrak{x}}}}, \sqrt[q]{1 - \left(1 - \prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(m_{i_\varrho}^u\right)^q\right)^{w_{i_\varrho}}\right)^{\frac{1}{C_n^\mathfrak{x}}}\right)^{\frac{1}{\mathfrak{x}}}}\right], \\
\left[\left(\sqrt[q]{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} \left(a_{i_\varrho}^\ell\right)^{\omega_{i_\varrho}}\right)^q\right)\right)^{\frac{1}{C_n^\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}, \left(\sqrt[q]{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} \left(a_{i_\varrho}^u\right)^{\omega_{i_\varrho}}\right)^q\right)\right)^{\frac{1}{C_n^\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}\right], \\
\left[\left(\sqrt[q]{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} \left(d_{i_\varrho}^\ell\right)^{\omega_{i_\varrho}}\right)^q\right)\right)^{\frac{1}{C_n^\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}, \left(\sqrt[q]{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} \left(d_{i_\varrho}^u\right)^{\omega_{i_\varrho}}\right)^q\right)\right)^{\frac{1}{C_n^\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}\right]
\end{array}\right)$$

Proof: By using Definition (9) we obtain:

$$\bigotimes_{\varrho=1}^{\mathfrak{x}} \left(\omega_{i_\varrho} \otimes P_{i_\varrho}\right) = \left(\begin{array}{c}
\left[\sqrt[q]{1 - \prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \left(m_{i_\varrho}^\ell\right)^q\right)^{\omega_{i_\varrho}}}, \sqrt[q]{1 - \prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \left(m_{i_\varrho}^u\right)^q\right)^{\omega_{i_\varrho}}}\right], \\
\left[\prod_{\varrho=1}^{\mathfrak{x}} \left(a_{i_\varrho}^\ell\right)^{\omega_{i_\varrho}}, \prod_{\varrho=1}^{\mathfrak{x}} \left(a_{i_\varrho}^u\right)^{\omega_{i_\varrho}}\right], \left[\prod_{\varrho=1}^{\mathfrak{x}} \left(d_{i_\varrho}^\ell\right)^{\omega_{i_\varrho}}, \prod_{\varrho=1}^{\mathfrak{x}} \left(d_{i_\varrho}^u\right)^{\omega_{i_\varrho}}\right]
\end{array}\right)$$

$$\left(\bigotimes_{\varrho=1}^{\mathfrak{x}}\left(\omega_{i_\varrho}\otimes P_{i_\varrho}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}} = \left(\begin{array}{c} \left[\left(\sqrt[q]{1-\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(m_{i_\varrho}^{\ell}\right)^{q}\right)^{\omega_{i_\varrho}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}, \left(\sqrt[q]{1-\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(m_{i_\varrho}^{u}\right)^{q}\right)^{\omega_{i_\varrho}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right], \\ \left[\sqrt[q]{1-\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\left(a_{i_\varrho}^{\ell}\right)^{\omega_{i_\varrho}}\right)^{q}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}, \sqrt[q]{1-\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\left(a_{i_\varrho}^{u}\right)^{\omega_{i_\varrho}}\right)^{q}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right], \\ \left[\sqrt[q]{1-\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\left(d_{i_\varrho}^{\ell}\right)^{\omega_{i_\varrho}}\right)^{q}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}, \sqrt[q]{1-\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\left(d_{i_\varrho}^{u}\right)^{\omega_{i_\varrho}}\right)^{q}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right] \end{array}\right)$$

$$\bigotimes_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(\bigotimes_{\varrho=1}^{\mathfrak{x}}\left(\omega_{i_\varrho}\otimes P_{i_\varrho}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}$$

$$= \left(\begin{array}{c} \left[\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(\sqrt[q]{1-\prod_{\substack{1\le i_1\le,,,\\<i_{\mathfrak{x}}\le n}}\left(1-\left(m_{i_\varrho}^{\ell}\right)^{q}\right)^{\omega_{i_\varrho}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}, \prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(\sqrt[q]{1-\prod_{\substack{1\le i_1\le,,,\\<i_{\mathfrak{x}}\le n}}\left(1-\left(m_{i_\varrho}^{u}\right)^{q}\right)^{\omega_{i_\varrho}}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right], \\ \left[\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\left(a_{i_\varrho}^{\ell}\right)^{\omega_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}, \sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\left(a_{i_\varrho}^{u}\right)^{\omega_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right], \\ \left[\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\left(d_{i_\varrho}^{\ell}\right)^{\omega_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}, \sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\left(d_{i_\varrho}^{u}\right)^{\omega_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right] \end{array}\right)$$

$$IVTSFWDMSM^{\mathfrak{x}}(P_1, P_2, P_3, \dots, P_n)$$

$$= \left(\begin{array}{c} \left[\sqrt[q]{1-\left(1-\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(m_{i_\varrho}^{\ell}\right)^{q}\right)^{\omega_{i_\varrho}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}}, \sqrt[q]{1-\left(1-\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\prod_{\varrho=1}^{\mathfrak{x}}\left(1-\left(m_{i_\varrho}^{u}\right)^{q}\right)^{\omega_{i_\varrho}}\right)^{\frac{1}{C_n^{\mathfrak{x}}}}\right)^{\frac{1}{\mathfrak{x}}}}\right], \\ \left[\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\left(a_{i_\varrho}^{\ell}\right)^{\omega_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}, \left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\left(a_{i_\varrho}^{u}\right)^{\omega_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}\right], \\ \left[\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\left(d_{i_\varrho}^{\ell}\right)^{\omega_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}, \left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_{\mathfrak{x}}\le n}}\left(1-\left(\prod_{\varrho=1}^{\mathfrak{x}}\left(d_{i_\varrho}^{u}\right)^{\omega_{i_\varrho}}\right)^{q}\right)\right)^{\frac{1}{C_n^{\mathfrak{x}}}}}\right)^{\frac{1}{\mathfrak{x}}}\right] \end{array}\right)$$

5. A Study of the Consequences of the IVTSFMSM Operator

In this section, some consequences of the MSM operators of IVTSFSs are discussed given some restrictions that show the usefulness and generalization of the proposed work.

Consider the IVTSFMSM operator.

$IVTSFMSM\,(P_1, P_2, P_3, \dots, P_n)$

$$= \left(\begin{array}{c} \left[\left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots \\ <i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x} m^{\ell}_{i_\varrho}\right)^{q}\right)\right)^{\frac{1}{C_n^x}}} \right)^{\frac{1}{x}}, \left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots \\ <i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x} m^{u}_{i_\varrho}\right)^{q}\right)\right)^{\frac{1}{C_n^x}}} \right)^{\frac{1}{x}} \right], \\ \left[\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots \\ <i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x}\left(1-\left(a^{\ell}_{i_\varrho}\right)^{q}\right)\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}}, \sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots \\ <i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x}\left(1-\left(a^{u}_{i_\varrho}\right)^{q}\right)\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}} \right], \\ \left[\sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots \\ <i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x}\left(1-\left(d^{\ell}_{i_\varrho}\right)^{q}\right)\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}}, \sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots \\ <i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x}\left(1-\left(d^{u}_{i_\varrho}\right)^{q}\right)\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}} \right] \end{array} \right)$$

1. For $m^{\ell} = m^{u} = m, a^{\ell} = a^{u} = a$, and $d^{u} = d^{\ell} = d$ the IVTSFMSM operator is reduced into MSM operators of TSFMSM.

TSFMSM $(P_1, P_2, P_3, \dots, P_n)$

$$= \left(\begin{array}{c} \left(\sqrt[q]{1-\left(\prod_{\substack{1\le i_1\le\cdots \\ <i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x} m_{i_\varrho}\right)^{q}\right)\right)^{\frac{1}{C_n^x}}} \right)^{\frac{1}{x}}, \sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots \\ <i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x}\left(1-\left(a_{i_\varrho}\right)^{q}\right)\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}} \\ \sqrt[q]{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots \\ <i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x}\left(1-\left(d_{i_\varrho}\right)^{q}\right)\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}} \end{array} \right)$$

2. For $= 2$, the IVTSFMSM operator is reduced into MSM operators of the IVSF environment.

$IVSFMSM\,(P_1, P_2, P_3, \dots, P_n)$

$$= \left(\begin{array}{c} \left[\left(\sqrt{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_x \le n}} \left(1 - \left(\prod_{\varrho=1}^{x} m_{i_\varrho}^{\ell} \right)^2 \right) \right)^{\frac{1}{C_n^x}}} \right)^{\frac{1}{x}}, \left(\sqrt{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_x \le n}} \left(1 - \left(\prod_{\varrho=1}^{x} m_{i_\varrho}^{u} \right)^2 \right) \right)^{\frac{1}{C_n^x}}} \right)^{\frac{1}{x}} \right], \\ \left[\sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_x \le n}} \left(1 - \left(\prod_{\varrho=1}^{x} \left(1 - \left(a_{i_\varrho}^{\ell} \right)^2 \right) \right) \right) \right)^{\frac{1}{C_n^x}} \right)^{\frac{1}{x}}}, \sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_x \le n}} \left(1 - \left(\prod_{\varrho=1}^{x} \left(1 - \left(a_{i_\varrho}^{u} \right)^2 \right) \right) \right) \right)^{\frac{1}{C_n^x}} \right)^{\frac{1}{x}}} \right], \\ \left[\sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_x \le n}} \left(1 - \left(\prod_{\varrho=1}^{x} \left(1 - \left(d_{i_\varrho}^{\ell} \right)^2 \right) \right) \right) \right)^{\frac{1}{C_n^x}} \right)^{\frac{1}{x}}}, \sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_x \le n}} \left(1 - \left(\prod_{\varrho=1}^{x} \left(1 - \left(d_{i_\varrho}^{u} \right)^2 \right) \right) \right) \right)^{\frac{1}{C_n^x}} \right)^{\frac{1}{x}}} \right] \end{array} \right)$$

3. For $q = 2$ and $a^{\ell} = a^{u} = a = 0$ the IVTSFMSM operator is reduced into MSM operators of the IVPyF environment.

$IVPyFMSM\,(P_1, P_2, P_3, \dots, P_n)$

$$= \left(\begin{array}{c} \left[\left(\sqrt{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_x \le n}} \left(1 - \left(\prod_{\varrho=1}^{x} m_{i_\varrho}^{\ell} \right)^2 \right) \right)^{\frac{1}{C_n^x}}} \right)^{\frac{1}{x}}, \left(\sqrt{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_x \le n}} \left(1 - \left(\prod_{\varrho=1}^{x} m_{i_\varrho}^{u} \right)^2 \right) \right)^{\frac{1}{C_n^x}}} \right)^{\frac{1}{x}} \right], \\ \left[\sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_x \le n}} \left(1 - \left(\prod_{\varrho=1}^{x} \left(1 - \left(d_{i_\varrho}^{\ell} \right)^2 \right) \right) \right) \right)^{\frac{1}{C_n^x}} \right)^{\frac{1}{x}}}, \sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_x \le n}} \left(1 - \left(\prod_{\varrho=1}^{x} \left(1 - \left(d_{i_\varrho}^{u} \right)^2 \right) \right) \right) \right)^{\frac{1}{C_n^x}} \right)^{\frac{1}{x}}} \right] \end{array} \right)$$

4. For $q = 1$, the IVTSFMSM operator is reduced into MSM operators of the IVPF set.

$IVPFMSM\,(P_1, P_2, P_3, \dots, P_n)$

$$
= \left(\begin{array}{c}
\left[\left(\sqrt{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\varkappa \le n}} \left(1 - \left(\prod_{=1}^{\varkappa} m^{\ell}_{i_\varrho} \right) \right) \right)^{\frac{1}{C_n^{\varkappa}}}} \right)^{\frac{1}{\varkappa}}, \left(\sqrt{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\varkappa \le n}} \left(1 - \left(\prod_{=1}^{\varkappa} m^{u}_{i_\varrho} \right) \right) \right)^{\frac{1}{C_n^{\varkappa}}}} \right)^{\frac{1}{\varkappa}} \right], \\
\left[\sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\varkappa \le n}} \left(1 - \left(\prod_{\varrho=1}^{\varkappa} \left(1 - \left(a^{\ell}_{i_\varrho} \right) \right) \right) \right) \right)^{\frac{1}{C_n^{\varkappa}}} \right)^{\frac{1}{\varkappa}}}, \sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\varkappa \le n}} \left(1 - \left(\prod_{\varrho=1}^{\varkappa} \left(1 - \left(a^{u}_{i_\varrho} \right) \right) \right) \right) \right)^{\frac{1}{C_n^{\varkappa}}} \right)^{\frac{1}{\varkappa}}} \right], \\
\left[\sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\varkappa \le n}} \left(1 - \left(\prod_{\varrho=1}^{\varkappa} \left(1 - \left(d^{\ell}_{i_\varrho} \right) \right) \right) \right) \right)^{\frac{1}{C_n^{\varkappa}}} \right)^{\frac{1}{\varkappa}}}, \sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\varkappa \le n}} \left(1 - \left(\prod_{\varrho=1}^{\varkappa} \left(1 - \left(d^{u}_{i_\varrho} \right) \right) \right) \right) \right)^{\frac{1}{C_n^{\varkappa}}} \right)^{\frac{1}{\varkappa}}} \right]
\end{array} \right)
$$

5. For $q = 1$ and $a^{\ell} = a^{u} = a = 0$, the IVTSFMSM operator is reduced into MSM operators of IVIF environment.

$IVIFMSM\,(P_1, P_2, P_3, \dots, P_n)$

$$
= \left(\begin{array}{c}
\left[\left(\sqrt{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\varkappa \le n}} \left(1 - \left(\prod_{\varrho=1}^{\varkappa} m^{\ell}_{i_\varrho} \right) \right) \right)^{\frac{1}{C_n^{\varkappa}}}} \right)^{\frac{1}{\varkappa}}, \left(\sqrt{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\varkappa \le n}} \left(1 - \left(\prod_{\varrho=1}^{\varkappa} m^{u}_{i_\varrho} \right) \right) \right)^{\frac{1}{C_n^{\varkappa}}}} \right)^{\frac{1}{\varkappa}} \right], \\
\left[\sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\varkappa \le n}} \left(1 - \left(\prod_{\varrho=1}^{\varkappa} \left(1 - \left(d^{\ell}_{i_\varrho} \right) \right) \right) \right) \right)^{\frac{1}{C_n^{\varkappa}}} \right)^{\frac{1}{\varkappa}}}, \sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\varkappa \le n}} \left(1 - \left(\prod_{\varrho=1}^{\varkappa} \left(1 - \left(d^{u}_{i_\varrho} \right) \right) \right) \right) \right)^{\frac{1}{C_n^{\varkappa}}} \right)^{\frac{1}{\varkappa}}} \right]
\end{array} \right)
$$

6. For $q = 1$ and $m^{\ell} = m^{u} = m$,$a^{\ell} = a^{u} = a = 0$ and $d^{\ell} = d^{u} = d$ the IVTSFMSM operator is reduced into MSM operators of IFS.

$IFMSM\,(P_1, P_2, P_3, \dots, P_n)$

$$
= \left(\left(\sqrt{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\varkappa \le n}} \left(1 - \left(\prod_{\varrho=1}^{\varkappa} m_{i_\varrho} \right) \right) \right)^{\frac{1}{C_n^{\varkappa}}}} \right)^{\frac{1}{\varkappa}}, \sqrt[q]{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\varkappa \le n}} \left(1 - \left(\prod_{\varrho=1}^{\varkappa} \left(1 - \left(d_{i_\varrho} \right)^{q} \right) \right) \right) \right)^{\frac{1}{C_n^{\varkappa}}} \right)^{\frac{1}{\varkappa}}} \right)
$$

7. For $q = 2$ and $m^{\ell} = m^{u} = m$, $a^{\ell} = a^{u} = a$, $d^{\ell} = d^{u} = d$ the IVTSFMSM operator is reduced into MSM operators of SFSs.

$SFMSM\ (P_1, P_2, P_3, \dots, P_n)$

$$= \left(\left(\sqrt{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} m_{i_\varrho} \right)^2 \right) \right)^{\frac{1}{C_n^{\mathfrak{x}}}}} \right)^{\frac{1}{\mathfrak{x}}}, \sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(a_{i_\varrho} \right)^2 \right) \right) \right) \right)^{\frac{1}{C_n^{\mathfrak{x}}}} \right)^{\frac{1}{\mathfrak{x}}}}, \sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(d_{i_\varrho} \right)^2 \right) \right) \right) \right)^{\frac{1}{C_n^{\mathfrak{x}}}} \right)^{\frac{1}{\mathfrak{x}}}} \right)$$

8. For $m^\ell = m^u = m, a^\ell = a^u = a = 0$, $d^\ell = d^u = d$ the IVTSFMSM operator is reduced into MSM operators of q-ROPFS.

$$QROPFMSM\ (P_1, P_2, P_3, \dots, P_n) = \left(\left(\sqrt[q]{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} m_{i_\varrho} \right)^q \right) \right)^{\frac{1}{C_n^{\mathfrak{x}}}}} \right)^{\frac{1}{\mathfrak{x}}}, \sqrt[q]{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(d_{i_\varrho} \right)^q \right) \right) \right) \right)^{\frac{1}{C_n^{\mathfrak{x}}}} \right)^{\frac{1}{\mathfrak{x}}}} \right)$$

9. For $a^\ell = a^u = a = 0$, the IVTSFMSM operator is reduced into MSM operators of IV q-ROPFS

$IVQROPFMSM\ (P_1, P_2, P_3, \dots, P_n)$

$$= \left(\left[\left(\sqrt{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} m_{i\varrho}^\ell \right) \right) \right)^{\frac{1}{C_n^{\mathfrak{x}}}}} \right)^{\frac{1}{\mathfrak{x}}}, \left(\sqrt{1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} m_{i\varrho}^u \right) \right) \right)^{\frac{1}{C_n^{\mathfrak{x}}}}} \right)^{\frac{1}{\mathfrak{x}}} \right], \left[\sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(d_{i\varrho}^\ell \right) \right) \right) \right) \right)^{\frac{1}{C_n^{\mathfrak{x}}}} \right)^{\frac{1}{\mathfrak{x}}}}, \sqrt{1 - \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ < i_\mathfrak{x} \le n}} \left(1 - \left(\prod_{\varrho=1}^{\mathfrak{x}} \left(1 - \left(d_{i\varrho}^u \right) \right) \right) \right) \right)^{\frac{1}{C_n^{\mathfrak{x}}}} \right)^{\frac{1}{\mathfrak{x}}}} \right] \right)$$

10. For $q = 1$ and $m^\ell = m^u = m$, $a^\ell = a^u = a$ and $d^\ell = d^u = d$ the IVTSFMSM operator is reduced into MSM operators of PFSs.

$$PFMSM\,(P_1, P_2, P_3, \dots, P_n) = \left(\begin{array}{c} \left(\sqrt{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x} m_{i_\varrho}\right)\right)\right)^{\frac{1}{C_n^x}}}\right)^{\frac{1}{x}}, \\ \sqrt{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x}\left(1-\left(a_{i_\varrho}\right)^q\right)\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}}, \sqrt{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x}\left(1-\left(d_{i_\varrho}\right)^q\right)\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}} \end{array} \right)$$

11. For $q = 2$ and $m^\ell = m^u = m, a^\ell = a^u = a = 0,\ d^\ell = d^u = d$ the IVTSFMSM operator is reduced into MSM operators of PyFSs.

$$PyFMSM\,(P_1, P_2, P_3, \dots, P_n) = \left(\begin{array}{c} \left(\sqrt{1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x} m_{i_\varrho}\right)^2\right)\right)^{\frac{1}{C_n^x}}}\right)^{\frac{1}{x}}, \\ , \sqrt{1-\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\<i_x\le n}}\left(1-\left(\prod_{\varrho=1}^{x}\left(1-\left(d_{i_\varrho}\right)^2\right)\right)\right)\right)^{\frac{1}{C_n^x}}\right)^{\frac{1}{x}}} \end{array} \right)$$

6. MADM Procedure Using IVTSFMSM Operators

In this investigation work, we develop a MADM procedure by using the idea of IVTSFMS, IVTSFWMSM, IVTSFDMSM, and IVTSFWDMSM operators based on IVTSFNs. To resolve the above types of issues, we choose the collection of alternative and their attributes whose expressions are summarized as $\mathfrak{D} = \{\mathfrak{D}_1, \mathfrak{D}_2, \dots, \mathfrak{D}_n\}$ denote the family of alternatives and $\bar{G} = \{\bar{G}_1, \bar{G}_2, \dots, \bar{G}_m\}$ denote the family of attributes. Moreover, the terms $\omega = \{\omega_1, \omega_2, \dots \omega_n\}$ represent the weight vector for the discussed attributes with a rule that is $\omega_\varrho \in [0,1], \varrho = 1,2, \dots n$ and $\sum_{\varrho=1}^{n} \omega_\varrho$. The information about the alternatives is taken in the form of IVTSFNs which are then subjected to the process of aggregation by using the proposed MSM operators. Complete steps of the MADM algorithm are given as follows:

Step 1: Develop a decision matrix for every alternative described by an IVTSFN under some attribute.

Step 2: To change cost type attributes to benefit type, the decision matrix obtained in Step 1 is normalized using the following equations.

$$t_{i_\varrho} = \left(\hat{m}_{i\varrho}, \hat{a}_{i\varrho}, \hat{d}_{i\varrho}\right) = \begin{cases} \left(m_{i\varrho}, a_{i\varrho}, d_{i\varrho}\right) & for\ benfits \\ \left(d_{i\varrho}, a_{i\varrho}, m_{i\varrho}\right) & for\ cost \end{cases}$$

Step 3: After normalization in Step 3, using the IVTSFMSM, IVTSFWMSM, IVTSFDMSM, and IVTSFWDMSM operators, we aggregate the IVTSF information.

Step 4: The aggregated information is analyzed utilizing a score function for ranking purposes.

Step 5: Based on the rules discussed in Definition 2, all the alternatives are ranked to obtain the optimum results.

To demonstrate the above-discussed steps, we present an example as follows:

Example 3: An educational institute wants to fulfill its vacant posts. It made a selection policy for the recruitment of the applicants. Four groups of alternatives (applicants) are $\{\mathfrak{D}_1, \mathfrak{D}_2, \mathfrak{D}_3, \mathfrak{D}_4, \mathfrak{D}_5\}$ selection of candidates according to the following four attributes defined by the competent authority $\{\bar{G}_1, \bar{G}_2, \bar{G}_3, \bar{G}_4\}$.

We adapt the same example to apply the MSM operators proposed in this paper where the information is based on IVTSFNs. We resolve a MAGDM problem for the selection of the most suitable candidates for their posts.

$\bar{G}_1$: Technical achievement.

$\bar{G}_2$: Potential market and market risk.
$\bar{G}_3$: Financial benefits.
$\bar{G}_4$: Development of science and employment creation.

For this, we consider the weight vector such as $\omega_\varrho = (0.3,0.1,0.4,0.2)^T$. Now, we present a stepwise computation of the above-discussed MADM problem using the algorithm discussed earlier in this section.

Step 1: All the technology enterprises are analyzed by a panel of decision-makers who gave their opinion using IVTSFNs to describe such enterprises in terms of the DM, DA, DNM, and DR. The provided information is given in Table 1 below.

Table 1. shows a decision maker given by the decision maker.

	G_1	G_2
B_1	([0.8,0.9],[0.6,0.7],[0.5,0.6])	([0.81,0.91],[0.61,0.71],[0.51,0.61])
B_2	([0.7,0.8],[0.5 0.6],[0.6 0.7])	([0.71,0.81],[0.51,0.61],[0.61 0.71])
B_3	([0.75,0.85],[0.4,0.5],[0.7,0.8])	([0.76,0.86],[0.41,0.51],[0.71,0.81])
B_4	([0.7,0.8],[0.1,0.2],[0.6,0.7])	([0.71,0.81],[0.11,0.21],[0.61,0.71])
B_5	([0.7,0.9],[0.2,0.5],[0.3,0.5])	([0.71,0.91],[0.21,0.51],[0.31,0.51])
	G_3	G_4
B_1	([0.82,0.92],[0.62,0.72],[0.52,0.62])	([0.83,0.93],[0.63,0.73],[0.53,0.63])
B_2	([0.72,0.82],[0.52,0.62],[0.62,0.72])	([0.73,0.83],[0.53,0.63],[0.63 0.73])
B_3	([0.77,0.87],[0.42 0.52],[0.72,0.82])	([0.78,0.88],[0.43,0.53],[0.73,0.83])
B_4	([0.72,0.82],[0.12,0.22],[0.62,0.72])	([0.73,0.83],[0.13,0.23],[0.63 0.73])
B_5	([0.72,0.92],[0.22,0.52],[0.32,0.52])	([0.73,0.93],[0.23,0.53],[0.33,0.53])

Step 2: The decision matrix given in Table 1 has all attributes of benefit type and does not need normalization, so we omit the step of normalization defined in the algorithm. In other words, after normalization, we get the decision matrix in Table 2 which is the same as given in Table 1.

Table 2. shows a transformation of a decision matrix into a normalization matrix.

	G_1	G_2
B_1	([0.8,0.9],[0.6,0.7],[0.5,0.6])	([0.81,0.91],[0.61,0.71],[0.51,0.61])
B_2	([0.7,0.8],[0.5 0.6],[0.6 0.7])	([0.71,0.81],[0.51,0.61],[0.61 0.71])
B_3	([0.75,0.85],[0.4,0.5],[0.7,0.8])	([0.76,0.86],[0.41,0.51],[0.71,0.81])
B_4	([0.7,0.8],[0.1,0.2],[0.6,0.7])	([0.71,0.81],[0.11,0.21],[0.61,0.71]
B_5	([0.7,0.9],[0.2,0.5],[0.3,0.5])	([0.71,0.91],[0.21,0.51],[0.31,0.51])
	G_3	G_4
B_1	([0.82,0.92],[0.62,0.72],[0.52,0.62])	([0.83,0.93],[0.63,0.73],[0.53,0.63])
B_2	([0.72,0.82],[0.52,0.62],[0.62,0.72])	([0.73,0.83],[0.53,0.63],[0.63 0.73])
B_3	([0.77,0.87],[0.42 0.52],[0.72,0.82])	([0.78,0.88],[0.43,0.53],[0.73,0.83])
B_4	([0.72,0.82],[0.12,0.22],[0.62,0.72])	([0.73,0.83],[0.13,0.23],[0.63 0.73])
B_5	([0.72,0.92],[0.22,0.52],[0.32,0.52])	([0.73,0.93],[0.23,0.53],[0.33,0.53])

Step 3: By using the IVTSFMSM, IVTSFWMSM, IVTSFDMSM, and IVTSFWDMSM operators, we aggregate the normalized decision matrix as follows in Table 3.

Table 3. shows the consequences of our proposed AOs by using the information in table 2.

Method	B_1	B_2	B_3
IVTSFMSM	([0.9082,0.9437], [0.979,0.9855], [0.971,0.979])	([0.8756,0.9052], [0.971,0.979], [0.979,0.9855])	([0.918,0.9232], [0.9625,0.9715], [0.9854,0.9914])
IVTSFWMSM	([0.88,0.9102], [0.9741,0.9799], [0.9667,0.9735])	([0.892,0.8738], [0.9667,0.9735], [0.9741,0.9799])	([0.8646,0.8905], [0.9577,0.9661], [0.9804,0.9858])
SFDMSM	([0.9912,0.9972], [0.8418,0.8718], [0.801,0.8378])	([0.9854,0.9914], [0.8041,0.8378], [0.8418,0.8718])	([0.9883,0.9943], [0.7604,0.8001], [0.8756,0.9052])
IVTSFWDMSM	([0.986,0.9922], [0.8167,0.8425], [0.7802,0.81])	([0.9804,0.9858], [0.8041,0.81], [0.8167,0.8425])	([0.9833,0.9889], [0.7399,0.7738], [0.8492,0.8738])
Method	D_1	D_2	
IVTSFMSM	([0.8756,0.9052], [0.9085,0.9351], [0.979,0.9855])	([0.8756,0.9437], [0.9351,0.9715], [0.9511,0.9715])	
IVTSFWMSM	([0.8492,0.8738], [0.904,0.9299], [0.9741,0.9799])	([0.8492,0.9102], [0.9305.0.9661], [0.9463,0.9661])	
IVTSFDMSM	([0.9854,0.9914], [0.5377,0.6363], [0.8418,0.8718])	([0.9854,0.9972], [0.6394,0.8001], [0.7077,0.8001])	
IVTSFWDMSM	([0.9804,0.9858], [0.5218,0.6153], [0.8167,0.8425])	([0.9804,0.9922], [0.6205 0.7738], [0.6867,0.7738])	

Step 4: By using Definition 2 of the score function, we examine the score values of the aggregated values in Table 4 as follows:

Table 2. contains score values of the proposed methodology by using the information in table 2.

	B_1	B_2	B_3	B_4	B_5
IVTSFMSM	-0.342	-0.233	-0.276	-0.098	-0.0126
IVTSFWMSM	-0.215	-0.149	-0.175	-0.054	-0.03
IVTSFDMSM	0.8287	0.7813	0.7551	0.9281	0.9841
IVTSFWDMSM	0.8903	0.8387	0.8305	0.9419	0.9667

Step 5: All the score values are analyzed and the ranking of the optimum technology enterprises is given in Table 4 below.

Table 5. shows the ranking and ordering of the score values.

Methods	Ranking Values
IVTSFMSM	$D_5 \geq D_4 \geq D_1 \geq D_2 \geq D_3$
IVTSFWMSM	$D_5 \geq D_4 \geq D_2 \geq D_3 \geq D_1$
IVTSFDMSM	$D_5 \geq D_4 \geq D_1 \geq D_2 \geq D_3$
IVTSFWDMSM	$D_5 \geq D_4 \geq D_1 \geq D_2 \geq D_3$

From the above discussions, we obtain the best alternative is D_5 by using the IVTSFMSM, IVTSFWMSM and IVTSFDMSM and IVTSFWDMSM operators. We also show the consequences of the score values in the following graphical representation of figure 1.

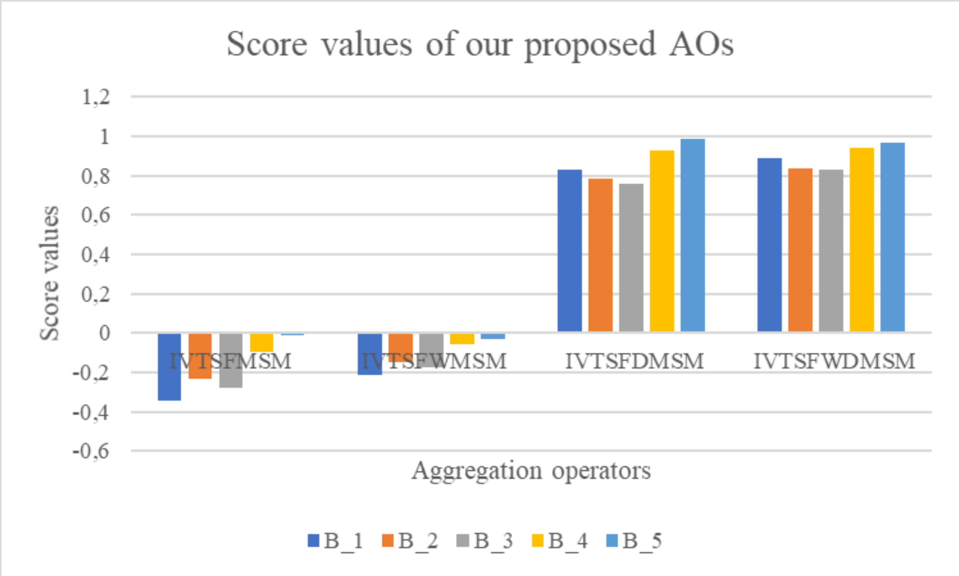

Figure 1. shows the score values of our proposed techniques.

7. Comparative Analysis

To evaluate the validity and competitiveness of the current proposed work based on IVTSFSs, we used some existing operators (Garg et al., 2017), (Zhang et al., 2014), (Liu et al., 2017), based on IVTSFSs to compare with proposed AOs. Improved interactive averaging AOs by using IVTSFNs by Garg et al. (2017), Einstein hybrid AOs of IVTSFNs by Zhang et al. (2014), and improved Hamacher AOs of IVTSFNs by Liu et al. (2017). The consequences of AOs shown in the following table 6 by utilizing the information of the decision matrix shown in table 1.

Table 3. shows the consequences of a comparative study with existing AOs.

Method	Score Values					Ranking Values
	$\dot{S}(D_1)$	$\dot{S}(D_2)$	$\dot{S}(D_3)$	$\dot{S}(D_4)$	$\dot{S}(D_5)$	
Garg et al. (2017)	-0.098	-0.278	-0.246	-0.058	0.0283	$D_5 \geq D_4 \geq D_1 \geq D_3 \geq D_2$
	-0.07	-0.177	-0.139	-0.047	-0.036	$D_5 \geq D_4 \geq D_1 \geq D_3 \geq D_2$
	0.7652	0.7265	0.6814	0.7856	0.87	$D_5 \geq D_4 \geq D_1 \geq D_2 \geq D_3$
	0.8565	0.7837	0.7328	0.8896	0.962	$D_5 \geq D_4 \geq D_1 \geq D_2 \geq D_3$
Zhang et al. (2014)	-0.069	-0.158	-0.122	-0.006	0.0089	$D_5 \geq D_4 \geq D_1 \geq D_3 \geq D_2$
	-0.106	-0.28	-0.25	-0.076	-0.052	$D_5 \geq D_4 \geq D_1 \geq D_3 \geq D_2$
	0.7437	0.6748	0.6646	0.8758	0.9526	$D_5 \geq D_4 \geq D_1 \geq D_2 \geq D_3$
	0.8676	0.843	0.8178	0.9171	0.9428	$D_5 \geq D_4 \geq D_1 \geq D_2 \geq D_3$
Liu et al. (2017)	-0.055	-0.150	-0.110	-0.0040	0.0355	$D_5 \geq D_4 \geq D_1 \geq D_3 \geq D_2$
	-0.098	-0.166	-0.134	-0.017	-0.006	$D_5 \geq D_4 \geq D_1 \geq D_3 \geq D_2$
	0.8634	0.7461	0.734	0.8287	0.9633	$D_5 \geq D_4 \geq D_1 \geq D_2 \geq D_3$
	0.8796	0.7826	0.6795	0.93444	0.9876	$D_5 \geq D_4 \geq D_1 \geq D_2 \geq D_3$
Proposed Operators	-0.34	-0.233	-0.276	-0.098	-0.0126	$D_5 \geq D_4 \geq D_1 \geq D_3 \geq D_2$
	-0.215	-0.149	-0.175	-0.054	-0.03	$D_5 \geq D_4 \geq D_1 \geq D_3 \geq D_2$
	0.8287	0.7813	0.7551	0.9281	0.9841	$D_5 \geq D_4 \geq D_1 \geq D_2 \geq D_3$
	0.8903	0.8387	0.8305	0.9419	0.9667	$D_5 \geq D_4 \geq D_1 \geq D_2 \geq D_3$

We observed from the consequences of existing AOs that $\mathfrak{D}_5$ is the best alternative. The geometrical representation is shown in Figure 2 by using the results of table 6.

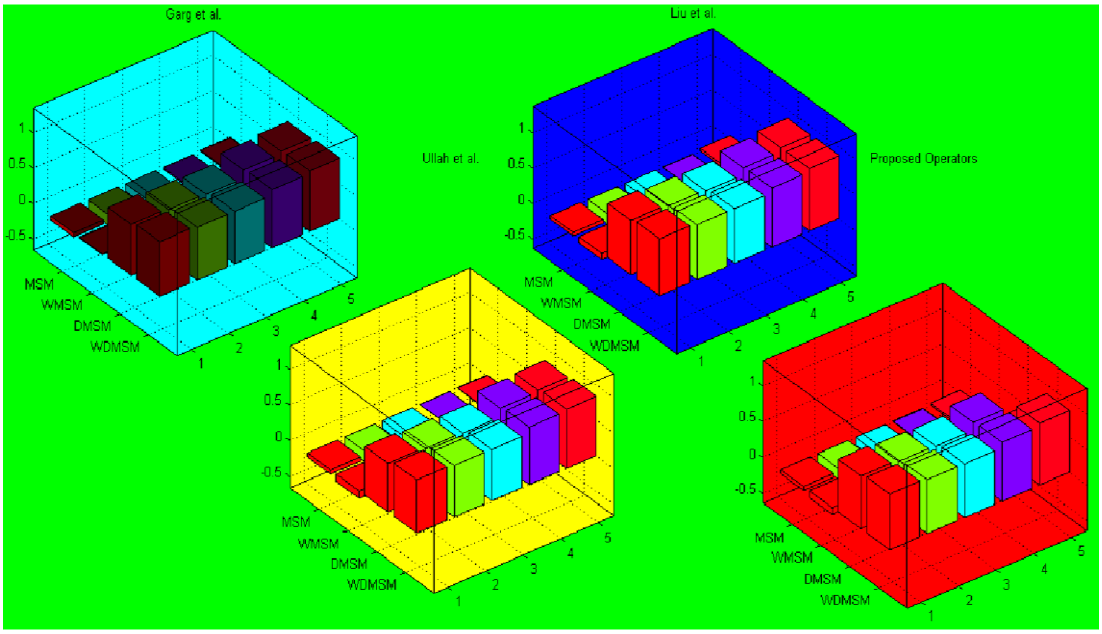

Figure 1. shows the results of the comparative study.

8. Conclusion

In this paper, we examined the conception of MSM operators in the environment of IVTSFSs as only the IVTSF environment provides flexibility in the assigning of the degrees of memberships and reduces information loss. Our main achievements are as follows:

1. We developed IVTSFMSM operators and IVTSFDMSM operators and investigated their characteristics.
2. To find the validity of our proposed AOs, we discuss some numerical examples.
3. The limitation of the previous work is observed and pointed out given some remarks where the generalization of the current work is proved.
4. The newly develop operators are utilized in the MADM problem to show their effectiveness and versatility.
5. The results obtained using IVTSFMSM operators are compared with the previous study.

We conclude two main advantages of our proposed work which are discussed as follows:

1. The use of fours degrees i.e., the DM, DA, DNM, and DR greatly reduces information loss which is very often in the frames of IVIFSs, IVPyFSs, and IVQROFSs.
2. The use of variable parameters gives us flexible boundaries in assigning the DM, DNM, DA, etc. unlike IVPFSs, IVIFSs, IVSFSs, and IVPyFSs.
3. In the future, we work on complex T Spherical fuzzy and interval-valued complex T spherical fuzzy.
4. In the coming future, we generalized our proposed work in the environment of complex IVTSF graphs (Hussain et al., 2021), (Ullah et al., 2022). Furthermore, we also enlarged our proposed work in the environment of PyF by using the Aczel Alsina operations (Hussain et al., 2022).

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