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Creation of a Genetic Algorithm to Locate the Optimal Position of Columns in a Regular Building

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ABSTRACT

The construction of buildings needs to consider a considerable number of variables and design rules to verify the structural integrity of the building. These rules require to consider the actions in the environment of the construction, the purpose of the building and the construction materials. The growing demand for taller and efficient buildings (safety rules and structural rules stricter) and the increasing prices of the construction's materials lead the engineers to find better ways to optimize the building for its propose and still complies all the structural rules. Thus, the use of optimization algorithms to accomplish a certain goal be usen more often. So, in this work we will use a Genetic Algorithm (GA) to determine a better position of columns in a regular and orthogonal building which the chosen goal is smaller. To accomplish this, we will use two different goals (weight and cost), two structural typologies (concrete and steel typology) and two different column positions methods. The experimental results indicate that it is possible to find good solutions but additional studies into the GA should be performed to increase the performance of the algorithm.

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1. Introduction

The building always had a huge importance in the society. They provide protection against the environment and comfort to its users. With the growing of the population, the constructions started being more constructed in height. Thus, this created new challenges to overcome. (Taranath, 1988)

However, there were being new improvements in terms of construction. New materials were created/improvement that will improve the building performance. The material production also was being improved which help increase the produce speed and quality and reduce the costs. (Teixeira et al., 2021)

Regarding the design methodologies, during the years new methodologies and others were improvement. This created faster and precise ways to calculate the structure which help to optimize the design process. Also, the design rules also were improvement. However, some of these methodologies needed a considerable number of calculations, which take a lot of time for the engineers. Additionally, these tasks were repetitive which can increase the probability generate an error.

With the use of computers lead a huge improvement in the civil engineering field. This creates the opportunity of have more calculation in shorter amount of time and less subject to human error. Thus, more complex or/and bigger analyses could be done (Stiffness matrix, FEM, FEA for example) in less time. With this improvement, and the improvements in the materials, more structures and more complex ones could be built. However, these building led to consider another factor. (Teixeira, 2020)

In the modern world, the cost of creation the building materials (structural and non-structural) and the erections of these materials is a key factor to take into consideration. In this way, the engineers try diverse ways to reduce the cost of the construction depending on the situation. One possibility is the position of the columns in the building.

So, in this paper will created an algorithm which, depending on the columns position, will perform the building design and calculate the respective building weight and cost. After, we will implement a GA to optimize the column position regarding two different objectives: Weight and Cost. Also, we will use two different alternatives of creating columns positions. Finally, we will perform calculations for two different typologies: Concrete and Steel.

The structure of this paper is organized as follows. In chapter 2 we check the most important woks concerning the GA and the optimization of buildings. After, in chapter 3 we will describe the considerations we assume for this article, mainly the fixed variable, the variables used by the GA and the fitness function. In chapter 4 we will present the setup and the analysis that will be perform and presented the results in chapter 5. Finally, in chapter 6 will draw the main conclusion and the future work.

2. State of Art

The use of optimization algorithms is openly being used the civil engineering field (Machairas et al., 2014; Teixeira, 2020). Also, there are several papers regarding the structure optimization when the sections profiles are the optimization variables.

Concerning trusses structure, (Gholizadeh, 2013; Li et al., 2009) with a Particle Swarm optimization (PSO) and (Kaveh & Talatahari, 2010; Pezeshk, 1997; Toğan & Daloğlu, 2006; Zhong et al., 2016) with a GA optimize the position and the topology of the truss elements to minimize the weight of the 2D and 3D structure. Both uses several approaches to represent the design variables and the evolutionary setup but most associates the design variable to the element. The work of (Zhong et al., 2016) uses two different matrices (topological and sizing) to optimize this two different criteria. Thus, the definition of the matrix needs to adapt to the problem. Also, beside the structural rules implemented, penalty factors to the fitness functions were added in other to simulate the restrictions.

The use of the GA is also used to optimize industrial buildings. The works of (H. K. Issa, 2010; H. Issa & Mohammad, 2008) focused in the minimization of the weight and also the minimization of the displacement to optimize the profiles in a fixed frame dimensions. In these works, the load and design were based on the rules from the Eurocodes and according to the BS 5950 and the model was constructed using the stiffness matrix.

In multistore structures, (Pezeshk, 1997) uses the symmetric properties and groups the structural elements in other to minimize the weight of a 2D Frame. (Khajeh et al., 2017) also optimize the profiles in different groups using the grid search method and a PSO. In these works, the load and the structure elements position were fixed. Additionally, (Nieto, 2016), does weight minimization using a GA in a 3D structure by using 3 different design standards. Here, it was used discrete design variables corresponding to the different steel profiles.

Regarding optimization of concrete structures, (Chan & Liu, 2000) uses a optimal criteria and a GA to minimize the structural elements weight and cost in tall buildings. In this paper, the design variables are the dimensions of the section element. The fitness implemented fitness function also have penalties for the restrictions regarding horizontal stiffness, differential deflections, and lateral drift.

About the optimization of columns positions, (Alencar Bandeira et al., 2022) uses a GA to minimize the sum of the total steel rebar weight and the volume concrete used of the building. Because the fitness function considers two different parameters, each parameter is multiplied by a normalization factor. The columns position is defined by a mesh when the algorithm selects random points representative of the mesh nodes. After, the combination of these points with the corner points generates multiples vertical and horizontal lines (or alignments), when each intersection of lines represents a column and the nodes the columns. Having the elements, the loads are assignment following the implemented rules. Then the model is executed by a FEM analysis. The beams design is made by considering the bending moments and the shear internal load. The columns design is made by an iterative searching for equilibrium of the section using Newton-Raphson method.

In most works, the connections between columns and beams are assumed as rigid. To study other types of connections, (Artar & Daloglu, 2015) implemented a GA algorithm to optimize the composite steel frames, where the weight is to minimize. To study the effect of this type of connections, (Csébfalvi & Csebfalvi, 2019) implement a GA to minimize the weight of the frame and the design respect the rules provided in the Structural Eurocodes. To calculate nodal force and flexibility was used a finite element program.

Given the importance of structure cost, (SARMA & ADELI, 2000) indicate the cost minimization should be an important factor to take into account in the optimization of a structure. This article also provides a method to calculate the respective cost of a building. The work from (Johnpaul, 2020) also present his method where he include the scheduling of the operations and implemented a GA to minimize the cost in a multi-story building. The work of (Kravanja & Žula, 2010) minimize the price of an industrial steel building but uses a MINLP strategy. Other works, (Bae & Horton, 2017; Tuhus-Dubrow & Krarti, 2010) focused more on the building life-cycle cost, where the building shape envelope is an important aspect to include.

3. Problem Considerations

In other to reach to goals proposed in this paper we created a simple Genetic Algorithm in other to find the best position of columns in a building that produces the minimal structural weight or the minimal cost of the building, depending on the chosen goal. Also, we will use two different typologies: Steel and Concrete. To accomplish this goal, the structural elements design is based on predesign databases that follow the rules presented in the Eurocodes. The loads assessment is based on fixed permanent loads (applied on the slabs and façade beams), variable loads depending on the structure environment, and considering simple supported beams for the load's distribution.

For the GA optimization, the variables contained the columns position needs to be converted to a chromosome. Because the slab orientation can influence the columns position, the orientation of the slabs also is a parameter that is included in the optimization. Additionally, there are other variables that will be use by the GA but need to be defined to characterize the problem environment.

3.1. Fixed variables

Regarding the environment variables, the user defines these. They characterize the building location, its shape, the presence of the bracing system or/and windows, soil, and actions. For this paper, the values considered are present below:

- Location: Porto, Portugal;
- Topography: Normal;
- Soil Type: Cohesive Soil, Firm Clays;
- Type of Structure: Steel Structure with Prefabricated Concrete Slab / Concrete Structure;
- Building: L shape building, with 25m maximum length and 35m maximum width;
- Windows's: It will be considered a window in all facades, excluding 0,5m from the corners. It will have 1,5 m in height and implemented 1m from the slab;
- Underground floors: 1 underground floor;
- Upper ground floors: 3 floors with 3m each
- Live Loads: Office load, Roof load (last floor) and parking loads (underground floor), No Fire
- Grid size: 1m x 1m

These variables are necessary to correctly define the structure and the values are the ones recommended by the Eurocode. Also, additional loads will be considered: The walls (non-structural) will be considered as a distribute load in the slab by 1.2 kN/m^2, the non-structural weight above the slab and below by 0,5 kN/m^2, the façade weight is considered as a distributed load in the facade beams equal to 2 kN/m.

Additionally, there are other considerations that where considered. In both structures we use a unidirectional prefabricated concrete slab (C25/30 and B500). In the case of steel structures, we use IPE profiles for the beams (S355), and HE profiles for the columns (S355). Also, the height and orientation of the column respects the beam width and the slab orientation.

In this paper, we did not consider a Bracing System. This means that the horizontal loads are not being considered once the structural materials only resist the vertical loads. In future works a bracing system algorithm will be implemented to the building being able to resist horizontal loads, like wind or seismic actions.

3.2. Optimization variables

As already mentioned, in the GA optimization the variables are the column position and the slabs orientation. To represent this column position in a chromosome it was considered the alignments position in a 2D top view of the building. Here, the intersection between two different orientation alignments produces a point that represents a column and the lines in each point represent the beams. In this study it will be used two different ways of creating these alignments. In both approaches, the facade alignments are mandatory.

In cases of different shapes besides rectangular, the building is subdivided into smaller rectangles in other to cover all area of the building. In these situations, the edges of these smaller rectangles are also mandatory. Because of having several rectangles, the alignments and orientations only are considered on the chromosome if the alignment are not defined previously by other rectangle. This is done to avoid alignment duplication on the gene once the alignments need to match between rectangles. In Figure 1 we presented a small example of a L shape building and an example of chromosomes, for both approaches of chromosome types allowed.

In the first approach, the chromosome is considered as binary type (see Figure 1 a). Here, each value represents if there is an alignment (1) or not (0) in the sequential position on the building, spaced by a grid value (indicated by the user). In the slab orientation, each value represents a direction (X or Y).

In the other approach, the chromosome is considered as an integer type (see Figure 1 b). In this situation, each value represents the number of additional alignments (besides the mandatories) in that direction that exist. This alternative does not provide alignment distances, so the alignments are equally spaced between them.

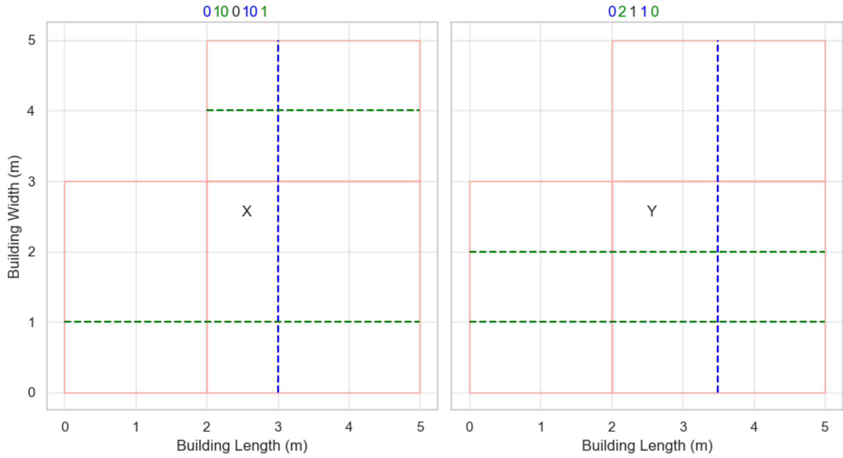

Figure 1. Examples of a Building genes. In red the mandatory alignments, blue the vertical alignments, green the horizontal alignments, grey the grid, and the text representing the slabs orientation: (a) Binary; (b) Integer

3.3. Fitness function

To perform the GA analysis two different ways of characterizing the building will be used: Structural Weight and Building Cost. These values are possible to calculate after the design of the building. To perform the design, first the algorithm creates the structural elements depending on the chromosome. Next, the respective loads are assembled and from the pre-design databases, the best profile that fulfills the imposed load and restrictions (concrete grade, steel grade, beside others). Then, in case the goal is to find the lighter structure, the fitness function is presented below in Eq. 1:

$$\text{f}(Ind) = \begin{cases} \sum_{i=0}^{n} w_i * L_i & if\ Ind\ is\ Valid \\ \infty & if\ Ind\ is\ not\ Valid \end{cases} \qquad (1)$$

When *Ind* is the Individual, w_i and L_i are the section weight and the length of the structural element (slabs, beams, columns, connections, and foundations) *i* of *n*.

In case the user selected the goal as cost, the fitness function is presented in Eq 2, when U_j and Q_j are the unit price and the quantity of the material *j* of *m* and T_e is the price of the task *e* of *l*. In the tasks is included the operation of the crane (for the erection of the materials) and it is calculated the total days of construction. In this calculation is consider the precedence of operations in the construction of the structural operations (Precedence Diagram Method).

$$\text{f}(Ind) = \begin{cases} \sum_{j=0}^{m} U_j * Q_j + \sum_{e=0}^{l} T_e & if\ Ind\ is\ Valid \\ \infty & if\ Ind\ is\ not\ Valid \end{cases} \qquad (2)$$

In some combinations of alignments, it is not possible to design a structure with the database used in the analysis. This happens where the load is big or/and the element length is big. Thus, in these cases it will not be possible to design the structure and calculate the fitness function. Thus, an infinite value is assigned to the fitness individual.

4. Analysis

In this paper it will be performed 8 different analyses with different assumptions for the structural typology, chromosome type and fitness goal. These analyses are presented in Table 1 where the GA parameters adopted are presented. For verification of the results authenticity, in each analysis was performed 30 different runs.

Table 1. GA's Parameters for the analyses with Concrete Typology and Steel Typology

Parameter	Settings A/E	Settings B/F	Settings C/G	Settings D/H
Initial Population	Random	Random	Random	Random
N. max of generations	200	100	200	100
Population size	20	20	20	20
Elite size	1	1	1	1
Tournament size	3	3	3	3
Crossover operator	Single point	Single point	Single point	Single point
Crossover rate	0.8	0.8	0.8	0.8
Mutation operator	Replacement	Replacement	Replacement	Replacement
Mutation rate	0.1	0.1	0.1	0.1
Fitness goal	Weight	Weight	Cost	Cost
Chromosome type	Binary	Integer	Binary	Integer
Structural Typology	Concrete/Steel	Concrete/Steel	Concrete/Steel	Concrete/Steel

Because of the analyses with an integer chromosome have a smaller domain of solutions, it is expected that the convergence will be faster. Thus, the maximum number of generations in the GA stopping conditions in these analyses is smaller.

5. Results

In the analyses *A* to *D* (Table 1) we evaluate the evolution (weight and cost) for the Concrete Typology and the analyses *E* to *H* we evaluate the evolution for the Steel Typology.

5.1. Concrete typology

In the Figure 2 is presented the GA evolution for the two different approaches for the chromosomes (Binary in analysis *A* and Integer in analysis *B*) for the goal weight. Here is possible to observe that in both approaches the structural weight could be minimize. Also, the several runs made for both analyses show that the divergence of results are relatively small. However, in analysis *A* the difference did not reduce in the last generations which means that not all solutions reach the same fitness value. In analysis *B* this difference between runs suggests that all runs reach the same fitness.

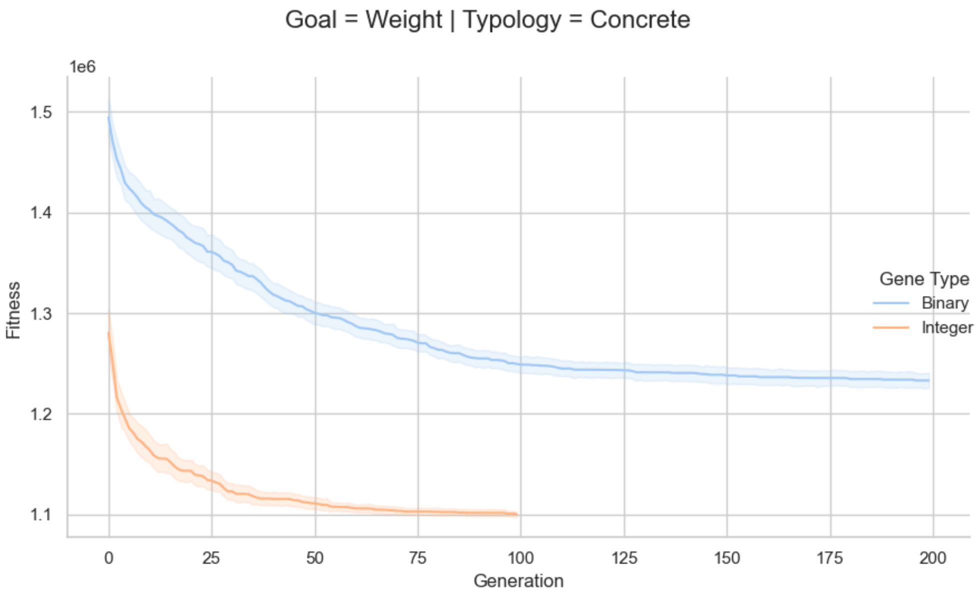

Figure 2. Evolution of the best fitness (Weight) for the Concrete Typology. Analysis A and B

Relative to the evolution, in the Figure 2 is possible to see that the evolution is faster in the first generations and gets slower during the generations in both approaches. However, from the results from the analysis *B* is possible to verify the analysis *A* possibility did not reach the best solution (Figure 3), although it may reach a local minimum. In Figure 3 (a) and Figure 3 (b) is possible to conclude that is true, once the alignments in Figure 3 (a) are not equally spaced. However, the solution in Figure 3 (b) cannot be replicated in (a) because the distance is not divisible by the grid.

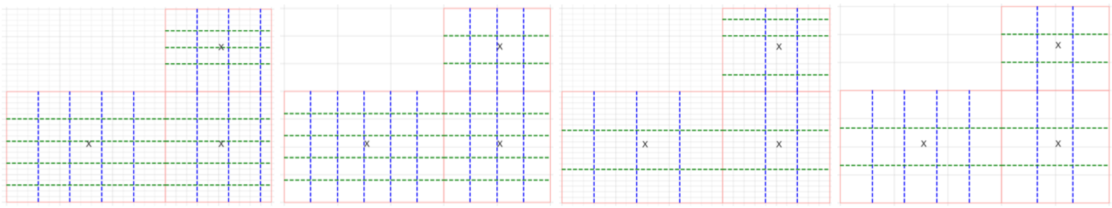

Figure 3. Alignments of the best analysis in Concrete typology: (a) Analysis *A*; (b) Analysis *B*; (c) Analysis *C*; (d) Analysis *D*

In the Figure 4 is presented the GA evolution for the analysis *C* and *D*, when the chromosome is assumed Binary and Integer for the goal cost. Here is possible to verify that the building cost was minimize after the GA reach the stopping conditions. Regarding the several runs in both approaches, although in the analysis *C* the difference is smaller, all runs have similarly the same behavior. However, in the analysis *C* the fitness between runs in the stopping generation are not the same. In the analysis *D* the difference is smaller, so most of the runs reach the same best fitness.

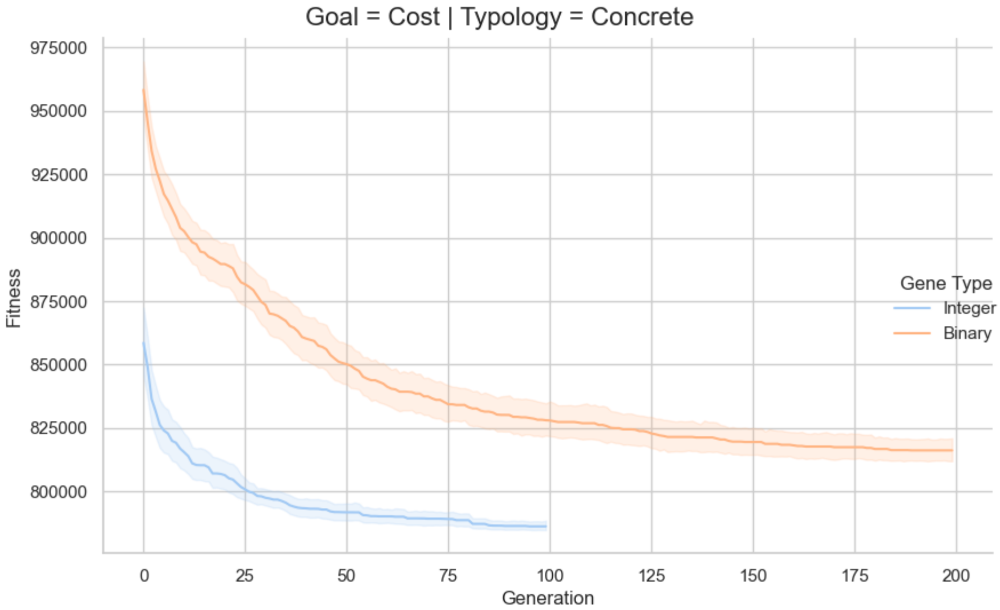

Figure 4. Evolution of the best fitness (Cost) for the Concrete Typology. Analysis C and D

Regarding the evolution, like the weight evolution (Figure 2), in the first's generations the best cost of the population decreased faster and then slower the latest stages of the evolution. Also, the analysis *C* possibility did not reach the best fitness comparing the best fitness from the analysis *D*, like show in Figure 3 (c) and Figure 3 (d). In this situation, the analysis *C* did not convert into the solution from the analysis *D*, although the solution has similarities. Is also noticeable in the Figure 3 that the spans for the best solutions with the goal cost (Analysis *C* and *D*) are bigger that the spans in the best solutions from the analyses *A* and *B* (goal weight).

5.2. Steel typology

For the case it is considered a steel typology, in Figure 5 is presented the GA evolution for the steel typology with the binary and integer types for the chromosomes (analyses *E* and *F*). With the Figure 5 was possible to minimize the structural weight for the steel typology. Also in both analyses, their behavior is similar in each run. However, in the analysis *G*, the difference between runs increased during the evolution and in the analysis *H* decreased. So, the analysis *G* did not converge to the same local minimum. That can be verified in the Figure 6 (a) and Figure 6 (b).

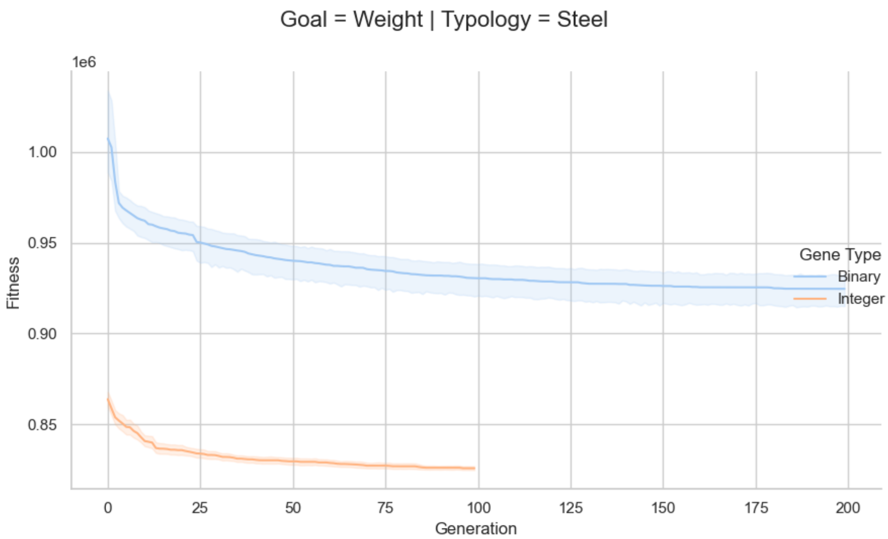

Figure 5. Evolution of the best fitness (Weight) for the Steel Typology. Analysis *E* and *F*

Regarding the evolution, it is noticeable that the best fitness value, in both approaches, decreased rapidly in the firsts generations and after that had a small but steady convergence util the stopping conditions. Additionally, the results for the analysis *E* possibly did not reach the best value, once the analysis *F* have better fitness value. However, because how the chromosome is created for this solution, can be outside the scope of the binary chromosome. This can be observed in the Figure 6, where the horizontal alignments in the Analysis *F* (Figure 6 a) cannot be replicated by the analysis *E* (Figure 6 b).

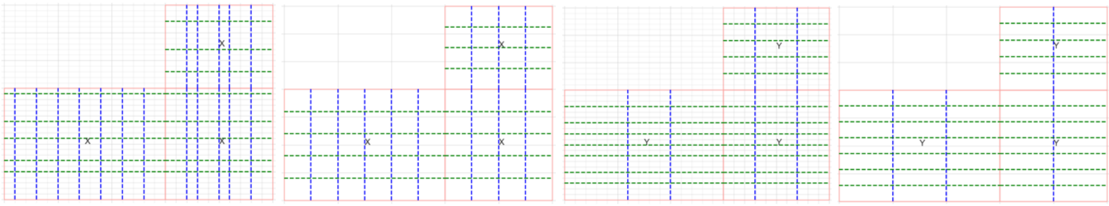

Figure 6. Alignments of the best analysis in Steel typology: (a) Analysis *E*; (b) Analysis *F*; (c) Analysis *G*; (d) Analysis *H*

For the GA evolution with the goal cost, is presented in the Figure 7 the minimization provided by the analyses *G* and *H*. Here is possible to verify that cost could be minimize. In both approaches, in the firsts generations the minimization was faster and then in close to the stopping conditions it was slower.

In the analysis *G* we can observe the difference between runs remain constant during the evolution, even in the last generation. So, the results did not converge to the same solution. Comparing to the evolution in the analysis *H*, the results in the analysis *G* could be improved. This is supported by the Figure 6 where is possible to observe the solution obtained by the analysis *H* (integer chromosome) is inside the domain of solutions if the chromosome were binary (analysis *G*).

In the analysis *H* it can be observed the difference between runs, where the first's generations the difference increased and then decreased at a constant rate. However, the results from both runs are not the same which means that the results did not converge to the same solution, even if the fitness are close.

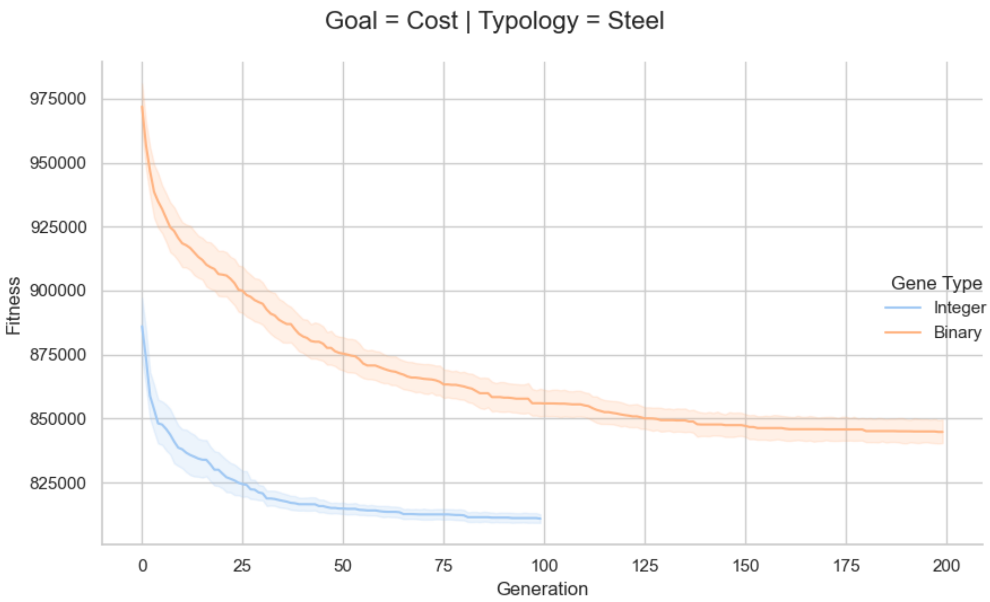

Figure 7. Evolution of the best fitness (Cost) for the Steel Typology. Analysis *G* and *H*

5.3. Results discussion

Analyzing both typologies is possible to observe that in all analyses the GA could minimize and provide good solutions. However, the solutions with binary chromosome did not reach the best solution. One reason is this type of chromosome generates a bigger domain of solutions (that also depends on the grid size) than the integer chromosome, so the GA needs more generations to converge into a satisfactory solution.

However, the integer approach can not adapt in case the structure has restrictions (that are not studied in this paper).

In the binary chromosome, the grid size is an important factor to considered in the results. It was possible to conclude that adding a smaller grid could improve the results. However, this change will increase the solution domain, and, in this way, the GA will need more time to converge into a satisfactory solution. Thus, the choice of this parameter needs to be careful considered when performing an analysis.

The distance between alignments is small comparing with the ones usually used in the real structure. This can be explained because of the methodology that is being used. The predesign tables and the loads assignment in the building are created based on a simple supported structure. Thus, the sagging moment of the beam is the conditional factor for the distance between alignments of the direction of the slab. Also, the created databases have a relatively small size to provide a good solution and not take too much computation time. So, this limitation can also interfere with the results. However, additional studies should be performed to analyze the impact of this issue. In the Figure 8 is presented the models of the best solutions obtained by the GA for the cost goal in the concrete (Figure 8 a) and steel typology (Figure 8 b).

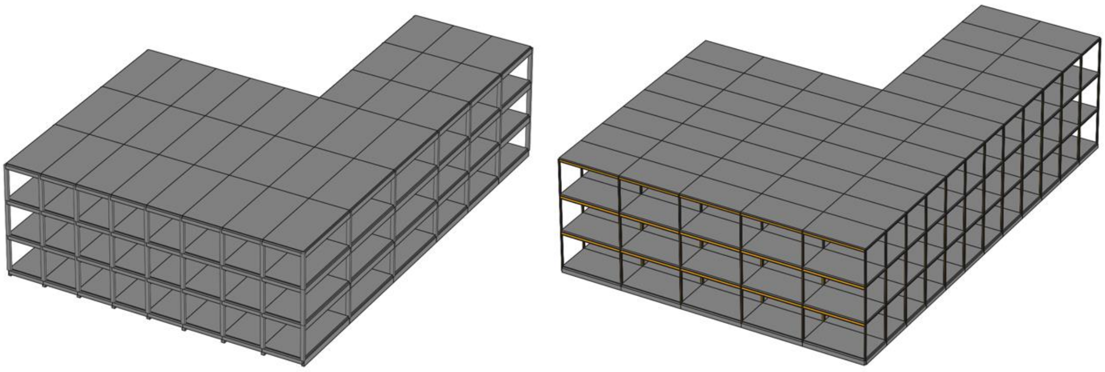

Figure 8. 3D Models for the best solution provided by the GA for cost (a) Analysis *D*; (b) Analysis *H*

Comparing the two objectives in both typologies, the analyses with cost as the GA goal were the ones which provided bigger spans. This is more noticeable in the steel structure, which provide bigger spans in the opposite direction of the slab orientation. Regarding the weight as the GA goal, the solutions are the same in the integer chromosome (analysis *B* and analysis *F*). In the binary chromosome, the solutions have some similarities, but we can conclude the GA did not reach and optimal solution. This is also noticeable in the solutions provided in the analyses *C* and *G*.

6. Conclusions

From the analysis perform in the different typologies (concrete and steel) we conclude that it was possible to implement a GA to obtain the best position of column to minimize the structural weight or the building cost. In this GA analyses was also possible that both types of chromosomes (binary and integer) used worked, but the integer type, provide better results than the binary type for this problem.

The implemented fitness function works in all range of possibilities created by the chromosome type used and by the fixed variables used to describe the building environment and building shape. Although it has some limitations (the structure is representative by a simple supported structure, do not support the horizontal actions, domain size, beside others) it can be representative of a behavior of a building. Additionally, the possibility of calculating the building cost can be extremely useful, once this parameter is truly relevant nowadays.

Observing the results from the analyses, it was possible to conclude the cost objective in the GA provide bigger spans in the solutions comparing to the goal weight. Also, using the goal cost, the steel typology provided bigger spans in the opposite direction than concrete typology, which provide alignments more similar between directions.

As for future works, we plan to add more functionalities in the fitness functions, as well the GA. In the fitness functions we plan to cover the horizontal load. Thus, adding a bracing system can be a possibility. Additionally, we plan to cover more structural typologies (composite for example) and add to include the fire action into the design.

In the GA we plan to add more options in the selection/pairing methods, as well the stopping conditions. After, we plan to make several parametric analyses with the objective to find the best GA setup parameters for the fitness function we have. Additionally, studies regarding the initial population can be performed to maximize the performance of the GA.

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