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Application of Electre Methods of Multi Criteria Decision Making in Systems with Missing Data

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ABSTRACT

In the paper, we consider the French school of decision-making approach, known as ELECTRE methods, in situations when some alternatives are not valuated with all criteria. For this purpose, we introduced a modified concordance index. Finally, we gave a practical example.

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1. Introduction

ELESTRE is a group of multicriteria decision-making methods that originated in Europe in the mid-1960s. ELECTRE is an acronym from the French word ELimination Et Choix Traduisant la REalité.

The French mathematician Bernard Rua and his colleagues at the consulting firm SEMA were the first to introduce this approach into decision theory (Кини & Райфа, 1981), (Roy, 1985). The need for such a system of multicriteria analysis emerged due to the risk that could not be avoided in classical methods in which the appropriate utility function was analyzed. Furthermore, ELECTRE methods essentially eliminate the subjectivity of decision-makers, and these methods became especially important after psychological research in the 1970s (Slovic, Kahneman, Tversky, 1982), (Larichev, 1990), which gave preference to qualitative methods.

ELECTRE methods later, due to their great applicability, developed in different directions, so variations of ELECTRE I, ELECTRE II, ELECTRE III, ELECTRE IV, ELECTRE IS, ELECTRE TRI and others arose.

In practice, we often encounter situations where some alternatives are not evaluated according to all criteria, i.e. there is a need to make decisions in systems with missing data. This problem often occurs in construction, as illustrated by the example in the fifth section.

In this paper, the modification of the ELECTRE method was introduced, which enables the problem of missing data to be elegantly overcome without diminishing the method's efficiency.

2. Basic Assumptions and Indications

Let us mark the alternatives we will study with:

$A_1, A_2, \ldots, A_n$.

Let them be evaluated according to criteria

$K_1, K_2, \ldots, K_m$

with weights

$\omega_1, \omega_2, \ldots, \omega_m$

and rating scales

$I_1, I_2, \ldots, I_m$.

Each of the rating scales is a subset of the set of natural numbers, and their diameters are:

$|I_1|, |I_2|, \ldots, |I_m|$.

With

$a_{1k}, a_{2k}, \ldots, a_{nk}$

We will mark the evaluations of alternatives according to the *k*-th criterion.

Let us now consider two alternatives with indices *i* and *j* and introduce the following notation:

$$K_{ij}^{+} = \{k \mid a_{ik} > a_{jk}\}$$

$$K_{ij}^{-} = \{k \mid a_{ik} < a_{jk}\}$$

$$K_{ij}^{=} = \{k \mid a_{ik} = a_{jk}\}$$

3. Concordance and Discordance Index

In the ELECTRE methods, the arguments favouring the hypothesis that alternative A is better than alternative B are contained in the so-called index of agreement with that hypothesis, i.e., the concordance index. In the original ELECTRE method, the concordance index considers a set of criteria according to which alternative A is not rated higher than alternative B.

So, let us have A_i i A_j alternatives that we compare. Then the index of agreement with the hypothesis that it is A_i better than A_j defined by equality.

$$C_{ij} = \frac{\sum_{k \in K_{ij}^{+} \cup K_{ij}^{=}} \omega_k}{\sum_{k=1}^{m} \omega_k}$$

What is new in the ELECTRE methods is the principle that a hypothesis for which a sufficiently large index of agreement is sufficient is not accepted if there is significant opposition to that hypothesis. The reasons for not accepting the hypothesis are the so-called index of disagreement or discordance.

If we test the hypothesis that an alternative A_i is better than an alternative A_j ,we define the index of disagreement with that alternative by the following relation.

$$d_{ij} = \max_{k \in K_{ij}^{-}} \frac{a_{jk} - a_{ik}}{|I_k|}$$

Let us also introduce the thresholds of agreement and disagreement. These are the numbers *p* and *q* such that

$$0 < q < p < 1.$$

We will say that the i-th alternative is better than the j-th if it is true that it is

$C_{ij} \geq p, \quad d_{ij} \leq q$

We will mark this with

$A_i \succ A_j$

In this way, we form a relationship between the alternatives.

4. Systems with Missing Data. Modification Aleksić - Anić

Often in practice, and especially in construction, we encounter the problem that some alternatives cannot be evaluated according to all the criteria under consideration, i.e. decisions should be made in systems with missing data.

ELECTRE methods are ideal for this type of problem because only the weights of the criteria are involved in calculating the compliance index, but not the scores on those criteria. Ratings are essential only for those criteria by which alternatives differ and only when calculating the discordance index. For example, suppose some comparing alternatives (or both) are not evaluated according to some criteria. In that case, we will calculate the indices as if the alternatives have the same evaluations according to those criteria.

Here is a modification of the primary ELECTRE method that allows working with missing data. We will introduce sets N_i representing the set of those indices k such that *i*-th alternative is not evaluated by *k*-th criterion. In this case, the agreement index is

$$C_{ij}^{N} = \frac{\sum\limits_{k \in K_{ij}^{+} \cup K_{ij}^{=} \cup N_i \cup N_j} \omega_k}{\sum\limits_{k=1}^{m} \omega_k}$$

The discrepancy index in case of missing data remains the same.

The concordance index introduced above can only be reduced by evaluating *i*–th alternatives according to the missing criteria. If we evaluate *j*–th alternative according to the missing criteria, the agreement index cannot increase again because those criteria have already been considered in the above sum. Thus, the concordance index is not increased by arbitrary criteria evaluation. In contrast, the non-compliance index can only be increased (the property of the maximum at a more extensive set). This modification has concluded that an unrated alternative will not be "handicapped" by not being evaluated or even because other alternatives are not evaluated according to the criteria by which it dominates. In other words, this consideration leads to the following conclusion:

Theorem (Aleksić, Anić): If in the modification of Aleksić-Anić the ELECTRE method the alternative is not found in the core, then in the arbitrary evaluation of alternatives according to criteria according to which these alternatives were not evaluated, they will not be found in the core. In other words, in a system with missing data, the core cannot be "poorer" than in applying the primary method if an arbitrary evaluation is performed.

The simplicity of overcoming the missing data problem is because we in the ELECTRE methods do not assign a global function to each alternative, as is done in quantitative methods. However, we do not consider other alternatives while comparing the two alternatives. Therefore, the relationship between two alternatives does not depend on the relationship between the other alternatives and their relation.

5. Example in Construction (Concrete Plant) with Missing Data

It is generally known that the conditions of business in construction are stochastic, subject to uncertainty and often with inaccurate or even data that in many cases are not known or missing. It is a consequence of many factors on which construction production, i.e. the construction process, depends, so it is not always possible to quantify all their measures (quantities/data) precisely. For example, data on temperature or other climatic influences over a long period are not always available (or are not available). Nevertheless, their trend prediction in the coming period could be made, which is essential for planning works (working days, possible winter, technological break).

For the needs of a specific example, a case is presented that is common in the everyday practice of realizing a construction investment. These are problems related to urban plans, specific locations, where projects take place. In the specific case, the table presents the lack of data on the long-term/longevity of the site in the sense that the site is permanent or temporary or that the building permit (building permit) is permanent or temporary. On the other hand, alternatives 3, 7, 8, 9 and 13 do not contain data on the site's longevity, which is a consequence of the uncertain procedure of adopting urban plans at the local government level and the often complicated administrative procedure. The missing data of environmental factors and environmental protection results from the untimely planning of urban utility infrastructure, i.e. collectors and landfills.

Name	w	Ik	Alt1	Alt2	Alt3	Alt4	Alt5	Alt6	Alt7	Alt8	Alt9	Alt10	Alt11	Alt12	Alt13	Alt14
Distance of the location from the construction site	10	0-4	4	3	2	0	1	2	4	0	3	1	4	3	0	1
Location access	8	1- 3	2	3	3	3	1	2	3	2	1	2	3	2	1	1
Location size	5	0-3	2	2	1	3	3	1	1	2	3	2	1	1	3	2
Possibility of prefabrication	5	0- 1	1	1	1	1	1	1	1	1	0	1	1	1	0	1
Infrastructure	7	0-3	3	2	2	2	1	3	2	3	3	2	2	1	3	2
Price of purchase and decoration	3	0-10	3	4	5	4	7	8	6	9	6	6	5	9	10	7
Location longevity	7	1-10	7	5		3	3	7				5	5	3		5
Environmental factor	8	0-2	1	2	2		2	2	2	2		1	2	2	2	

If now in the modified ELECTRE method for the thresholds of agreement or disagreement we take p = 0.75, q = 0.3, then the relationship between the alternatives is shown in Figure 1. We can see that there is only one alternative left in the kernel - alternative 1.

Figure 2 shows a graph of the relationship between the alternatives in the case of the larger parameter p = 0.8. The core of alternatives is now getting more prosperous and consists of alternatives 1, 6 and 9.

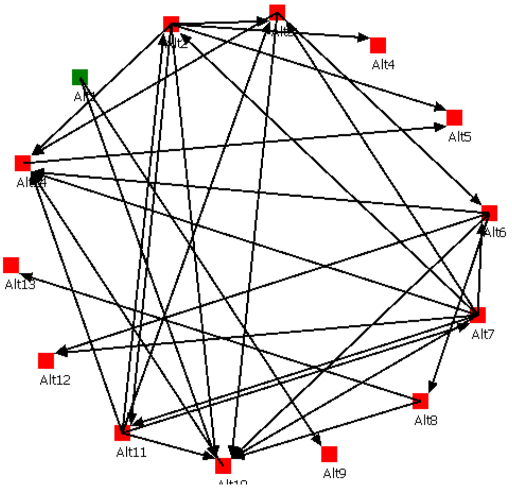

Figure 1.

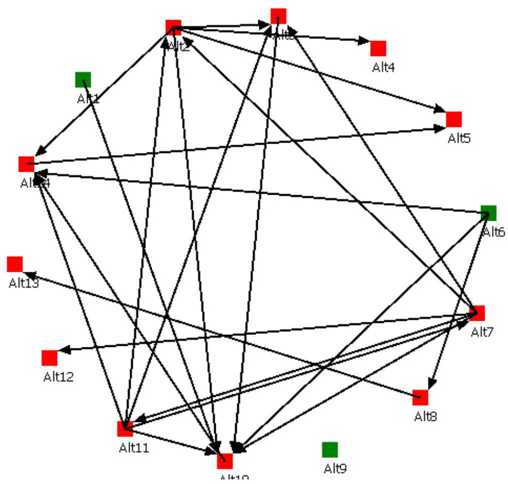

Figure 2.

6. CONCLUSION

The ELECTRE method described here, as well as other ELECTRE methods, separates the core from a multitude of alternatives, in which all alternatives are incomparable - they are considered equally good. More precisely, we do not have enough reason to be able to throw some of the alternatives out of the core.

Human errors in determining the weight of criteria and rating scales are a severe problem. ELECTRE methods partially eliminate this problem, and in the business plan, this problem is reduced by collective decision-making through assemblies, boards of directors, board of directors.

What is new in this paper is that we are dealing with missing data for the first time in the ELECTRE methods, which is often the case in construction, as the example shows.

For this purpose, a new concordance index is introduced, which does not reduce the method's efficiency, which is shown in this paper.

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