Operations Research and Engineering Letters
Vol. 5, No. 1, 2026, pp. 1-13
ISSN: 2812-9318

Determination of Formation Constants by a Step Drawdown Test at the "University City" Site in Banja Luka

Milan Jakšić [1], Žana Topalović [1], Nataša Popović-Miletić [1], Anica Milanović [1], Petar Praštalo [1]
[1] University of Banja Luka, Faculty of Architecture, Civil Engineering and Geodesy, e-mail: info@aggf.unibl.org

Article Info

Article history:

Received Jun 3, 2026
Revised Jul 13, 2026
Accepted Jul 23, 2026

Keywords:

Unconfined aquifer
Delayed yield
Step drawdown test
Simplified numerical method

ABSTRACT

This paper presents hydrogeological characterization of the alluvial aquifer at the "University City" site on the left bank of the Vrbas River in Banja Luka, Bosnia and Hercegovina. A step drawdown test with three successively increasing pumping rates (1.0, 1.8, and 2.5 l/s) was carried out at well B3, with groundwater-level drawdown and recovery monitored at well B3 and piezometer P1 over 34.7 hours using automatic dataloggers. The time-drawdown curve at P1 exhibits the characteristic "S" shape form of an unconfined aquifer with delayed gravity yield: an early steep segment reflecting confined-type storage release, an intermediate flattening caused by slow gravity drainage, and a late steep segment approaching the Theis curve. Aquifer type was identified from the shape of the curve, and preliminary formation constants were obtained by Neuman type-curve matching of the time-drawdown data for the first pumping rate. Final parameter values were refined by fitting observed drawdowns and recovery at all three pumping rates simultaneously, with calculated drawdowns from the numerical model based on principle of superposition („Inverse procedure"). The resulting formation constants are: transmissivity $T = 7.08–7.20 \times 10^{-3}$ m²/s; specific yield $S_y = 0.125–0.140$; elastic storage coefficient $S = 0.013–0.020$; specific storage $S_S = 2.54–3.91 \times 10^{-3}$ 1/m; horizontal hydraulic conductivity $K_H = 1.39–1.41 \times 10^{-3}$ m/s; vertical hydraulic conductivity $K_V = 2.58–2.62 \times 10^{-4}$ m/s; and anisotropy ratio $k_D \approx 0.186$. Well-loss coefficients for B3 are $A_1 = 2.6$ (linear) and $B = 9500$ (quadratic). The number of the corresponding type curve is $\beta = r/B = 0.4$, delayed-yield index is $1/\alpha \approx 600$ s, and the influence of delayed yield on drawdown ceases after approximately 30 minutes of pumping. The results provide the first baseline formation constants for this stretch of the Vrbas alluvium and demonstrate that a step drawdown test, interpreted with type curves method and method of inverse procedure, can yield consistent unconfined-aquifer parameters from one pumping well and a single observation piezometer.

Copyright H© 2026 Faculty of Civil Engineering Management, University "UNION–Nikola Tesla", Belgrade, Serbia.
All rights reserved.

Corresponding Author:

Milan Jakšić,
University of Banja Luka, Faculty of Architecture, Civil Engineering and Geodesy, Bosnia and Hercegovina.
Email: milan.jaksic@aggf.unibl.org

1. Introduction

Alluvial aquifers along major river courses supply drinking water to a large proportion of urban populations in the Western Balkans. Their hydraulic characterization is complicated by the fact that they are frequently unconfined, and their response to pumping is governed by delayed gravity yield from the unsaturated or

partially saturated layers above the depression cone. When such an aquifer is pumped, the time–drawdown curve does not follow the Theis solution for a confined aquifer, nor does it follow the simple unconfined response immediately. Instead, it passes through three characteristic segments: an early steep segment in which the aquifer reacts as if confined and water is released from storage by compressibility of the pore water and the aquifer skeleton; an intermediate flattening caused by slow gravity drainage from the pores above the depression cone; and a late steep segment in which the drawdown again follows the Theis curve, this time with the specific yield Sy controlling the response (Boulton, 1963) (Prickett, 1965) (Walton, 1960).

This behaviour was first described mathematically by Boulton (1963), who introduced the concept of delayed yield from storage, and was subsequently reformulated by Streltsova (1972) and Gambolati (1976). The most widely used interpretation framework, however, is that of Neuman (1975), who developed an analytical solution and a family of type curves for anisotropic unconfined aquifers considering delayed gravity yield. Boulton's and Neuman's graphical methods have been standard practice, and are reproduced in the principal reference works on pumping-test analysis (Prickett, 1965) (Kruseman & de Ridder, 1994). The central limitation of the graphical method for applied work is that it assumes a constant pumping rate from the beginning of the test. In practice, step drawdown tests, in which the pumping rate is increased in two or more discrete steps, are often preferred, because they allow simultaneous estimation of aquifer parameters and well-loss coefficients from a single test, and because they avoid the operational difficulties of maintaining a strictly constant rate over many hours.

Step drawdown tests are routinely analysed by the methods of Hantush & Jacob (1955) and Rorabaugh (1953), which assume confined or leaky aquifer behaviour and separate aquifer loss from well loss empirically. These methods are not directly applicable to unconfined aquifers with delayed gravity yield, because the aquifer loss term itself varies with time in a way that depends on the transition from elastic storage release to gravity drainage. Where type curves are used for step-test data, the usual approach is to analyse each step separately as if it were a constant-rate test, which introduces errors both because the drawdown at the start of each step carries over from the previous step, and because the storage mechanism active during the test may differ between steps. A more consistent approach is to apply the principle of superposition to the variable-rate history and to fit the observed drawdown and recovery simultaneously by adjusting the aquifer parameters. This approach is mathematically straightforward but is surprisingly under-documented in the applied literature for unconfined aquifers with delayed yield, particularly where a single test is used to characterize both the aquifer and the pumping well.

The present study concerns the "University City" site in Banja Luka, Republika Srpska, situated on the left-bank alluvial terrace of the Vrbas River. Three production wells (B1, B2, and B3) have been in operation at the site for many years, and three piezometers (P1, P2, and P3) were recently installed as part of a Ministry-funded project on the water balance of surface and groundwater in the area. Despite the operational importance of the wells, no pumping test has ever been carried out in this area, and neither the aquifer type nor its formation constants have been documented in the published literature. This is the first hydrogeological characterization of the site, and it provides baseline data required for the ongoing water-balance assessment.

The objectives of this paper are therefore threefold: (i) to identify the type of aquifer at the University City site from the shape of the time–drawdown curve obtained during a step drawdown test; (ii) to estimate the formation constants of the aquifer i.e. transmissivity, specific storage, specific yield, horizontal and vertical hydraulic conductivity and anisotropy, from drawdown and recovery data collected at well B3 and piezometer P1; and (iii) to evaluate whether a superposition-based interpretation of the step-test data, using standard type-curve values as a starting point, can reproduce the observed response at all three pumping rates and during recovery. The novelty of the work lies in providing the first formation constants for this stretch of the Vrbas alluvium and in demonstrating that a single step drawdown test, monitored at one well and a single piezometer, can yield a consistent set of unconfined-aquifer parameters when analysed with this combined approach.

2. Site description, research materials and methods

2.1. Site description and hydrogeological setting

The "University City" site is located in Banja Luka, Republika Srpska, Bosnia and Herzegovina, on the left-bank alluvial terrace of the Vrbas River (Figure 1). The investigated area covers approximately 28 ha and comprises the university campus and its immediate surroundings. The terrain is flat to gently sloping, with ground elevations between approximately 156.1 and 156.5 m above sea level.

The regional geological setting is defined by the Banja Luka sheet of the Basic Geological Map of the former Yugoslavia (Mojičević et al., 1969). The site is underlain by Quaternary alluvial deposits of the Vrbas River, consisting of interbedded sands, gravels, and sandy gravels with occasional silty and clayey

intercalations. These deposits form an unconfined aquifer whose water table is hydraulically connected to the Vrbas River. The alluvial sequence is underlain by impermeable Miocene marls and clays, which form the base of the aquifer. The thickness of the alluvial sequence at the site, determined from borehole logs during piezometer installation, is from 8 to 10 m.

The direction of groundwater flow at the site is approximately perpendicular to the river bed, from the campus towards the Vrbas. Three existing production wells (B1, B2, and B3) are aligned parallel to the river bank and supply water from the alluvial aquifer. Well B3 was selected as the pumping well for the present study because of its favourable position relative to the observation points and its available pumping capacity.

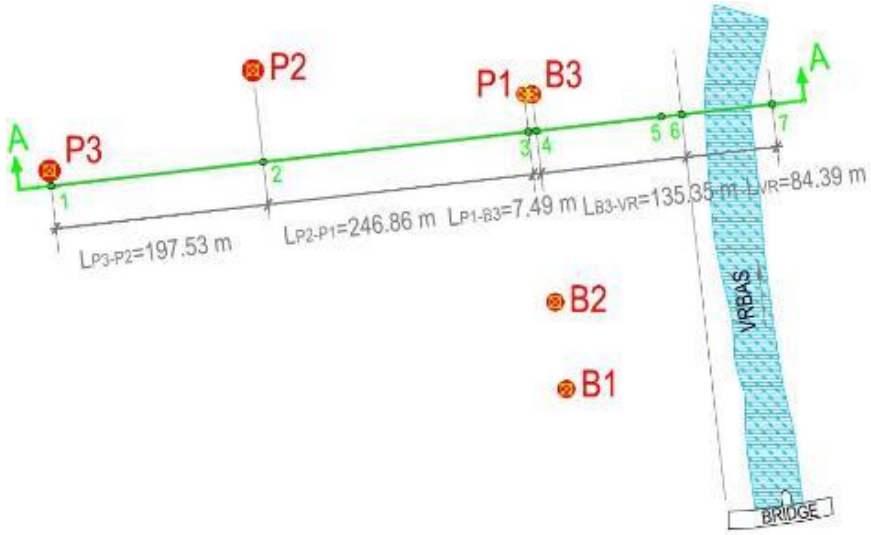
Three piezometers (P1, P2, and P3) were installed as part of a Ministry-funded project on the water balance of surface and groundwater in the area. Their locations were chosen so that they are aligned approximately perpendicular to the river bed, following the inferred direction of groundwater flow. Piezometer P1 is located closest to the river, next to the existing well B3; piezometers P2 and P3 are located at progressively greater distances from the river (Figure 1). This arrangement allows the spatial variation of drawdown to be monitored during pumping.



Figure 1. Layout of the area considered.

The soil profile was constructed along a transect A–A passing through the piezometer locations and oriented perpendicular to the direction of river bed. Because transect A–A intersects the river bed approximately 370 m downstream of the bridge where an ADCP cross-section measurement was done, the profile at the intersection has been reconstructed, geometrically and elevation-wise, from the measured cross-section taking into account an average bed slope of Id = 5‰. The resulting profile is shown in Figure 2, and confirms the lithological sequence described above.

(a)



(b)

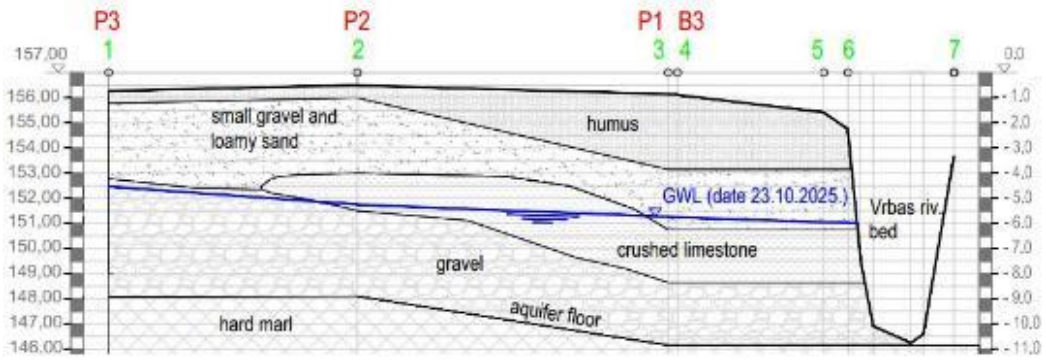


Figure 2. (a) Layot of transect A-A, (b) Soil profile along transect.

2.2. Borehole and piezometer construction

The piezometers were drilled on 17–19 July 2025. using the spiral drilling method. All boreholes reached the base of the alluvial aquifer. Piezometer P1 has a diameter of 75 mm and a depth of 10.0 m and is constructed of PVC pipe; piezometers P2 and P3 have a diameter of 50 mm and depths of 8.3 m and 8.1 m respectively and are constructed of galvanized steel pipe. Each piezometer is fitted with a lockable cap. Pressure transducers, each connected to optical cable of 15 m in length, were hanged on these caps from inside of piezometer tubes.

During drilling works, the type and thickness of each soil layer were recorded by visual inspection of the extracted material and by the method of instantaneous drilling core length, and a core sample was taken at every change of lithology. The resulting logs confirmed the sequence of sandy gravels with silty intercalations described in Section 2.1 and showed that the base of the aquifer was reached at every piezometer location.

Geodetic measurement of the position and elevation of the tops of the piezometer pipes, of the top of well B3, and of the ground surface at each location was carried out in October 2025. The absolute elevations of the pipe crowns and of the ground at each object are given in Table 1. These elevations were used to convert all measured water levels to a common datum.

Table 1. Absolute elevations of points on objects

Piezometer/well	Pipe crown elevation	Ground elevation
B3	155.117	156.117
P1	156.450	156.146
P2	156.810	156.496
P3	156.660	156.281

2.3. Step drawdown test: design, instrumentation, and implementation

Before and after the step drawdown test, groundwater levels were measured manually in all three piezometers once per day on several occasions, in order to establish the static water level (SWL) at the site. Measurements were made with a level meter equipped with audible and visual signal. The results are given in Table 2.

Table 2. Static levels in piezometers

Before test			
Date	P3	P2	P1
13.10.2025.	152.49	151.76	151.25
14.10.2025.	152.49	151.76	151.25
15.10.2025	152.48	151.75	151.25
16.10.2025.	152.49	151.75	151.27
After test			
23.10.2025.	152.47	151.75	151.26
03.12.2025.	152.86	152.11	151.70
09.12.2025.	152.79	152.05	151.63

The step drawdown test was carried out on 20–21 October 2025. It consisted of three consecutive pumping steps with successively increasing rates, followed by a recovery period after the pump was switched off. The total duration of the test, from starting of the pump to the moment when groundwater returned to the static level, was 2,082 min (34.7 h) at piezometer P1 and 2,076 min (34.6 h) at well B3. The pumping rates and the times at which they were changed are given in Table 3.

Table 3. Data on flow rate values and timing of flow rate changes during the pumping test

Date	Time	Flow rate (l/s)
20.10.2025.	10:58	1.0÷1.2
20.10.2025.	18:30	1.8÷2.0
21.10.2025.	10:30	2.5÷2.7
21.10.2025.	18:08	0.0
21.10.2025.	21:40	Groundwater static level

Well B3 was used as the pumping well. It consists of a water shaft constructed above the well pipe, with a well-construction diameter of 110 mm and a total depth of 11 m including the settling tank. The well is equipped with a Pedrollo 4Block m6/9 submersible pump (1.5 hp), with a nominal operating range of Q = 0–2.5 l/s and H = 61–26 m. The pump discharge is connected to a pressure PE pipeline that terminates in an arched threaded steel fitting embedded in the upper concrete slab of the well shaft.

The pumped water was conveyed away from the zone of influence of the well by a fire hose of 50 mm diameter and 85 m length. One end of the hose was connected to a ball valve at the well shaft; the other end discharged beyond a fence line parallel to the Vrbas River, at a sufficient distance that no recirculation to the aquifer could occur during the test (Figure 3d).

The first pumping rate was measured by the volumetric method, using a 70-litre barrel and a stopwatch. The second and third rates were measured with a Panda PUTF 205 ultrasonic flow meter (Figure 3a). The measured rates were Q_1 = 1.0 - 1.2 l/s, Q_2 = 1.8 - 2.0 l/s, and Q_3 = 2.5 - 2.7 l/s. The times of the pumping rate and of increase in pumping flow are given in Table 3.

Groundwater levels were monitored at piezometer P1 (Figure 3c) and at well B3, using automatic pressure transducers (TD Diver and Baro Diver, Series DI8, van Essen Instruments), with a recording interval of 1 min (Figure 3b). A Baro Diver was used to record barometric pressure for compensation of the pressure records. Manual measurements with the groundwater level meter were used to check the transducer records at the beginning and end of the test (Figure 3d).

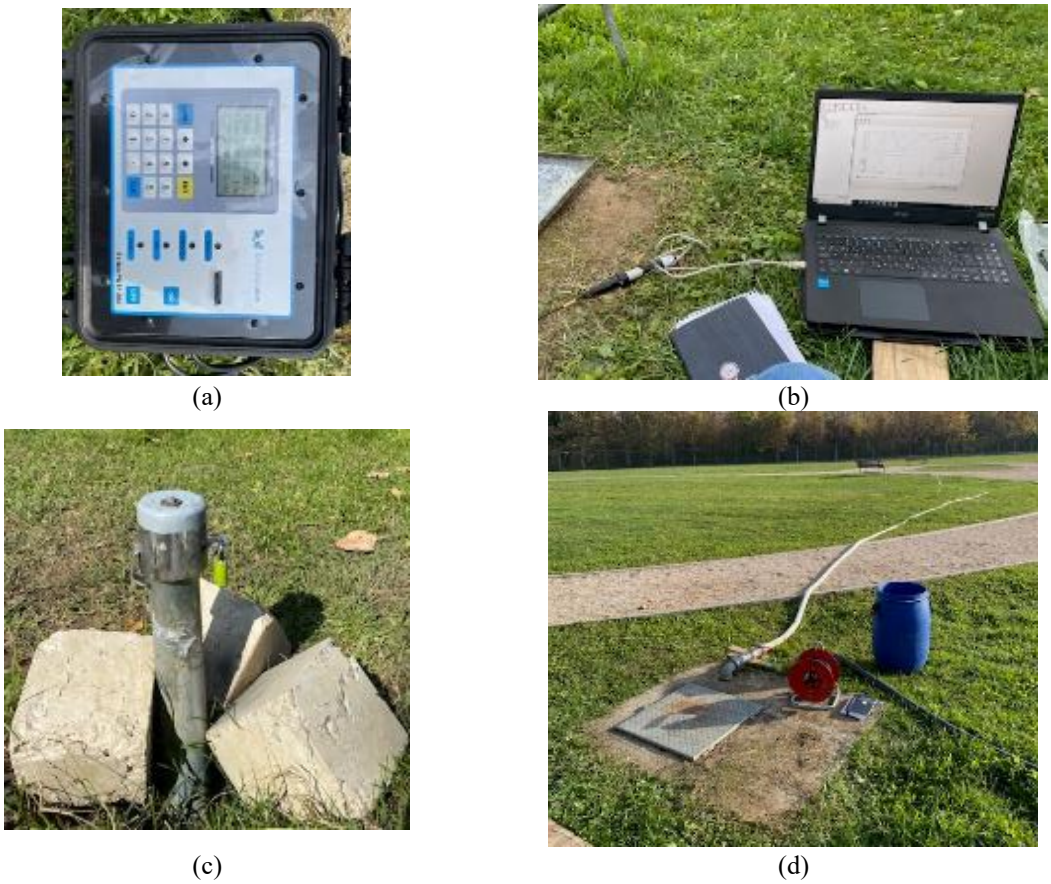
2.4. Data processing methods

Drawdown data from the test were processed in three stages. First, the aquifer type was identified from the shape of the time–drawdown curve at piezometer P1 plotted on a log–log scale, for the first pumping rate. Second, preliminary aquifer parameters were estimated by matching the time-drawdown data to the Neuman's type curves for an unconfined aquifer with delayed gravity yield. Third, because the Neuman method assumes a constant pumping rate, final parameter values were obtained by an inverse, superposition-based, procedure

in which the observed drawdowns and recovery at all three pumping rates were fitted with calculated drawdowns from simplified numerical model. The two procedures are described below.

The Neuman type-curve method uses two families of curves: the Type A curves, which describe the early, confined-type response, and the Type B curves, which describe the late, unconfined response. Each pair of Type A and Type B curves shares a common horizontal asymptote and is identified by a single value of the parameter $\beta = r/B$, where r is the radial distance from the pumping well to the observation point and B is the leakance or delayed-yield characteristic length. The spacing between the Type A and Type B curves is governed by the ratio $\sigma = S/Sy$.

Following the constraint of Neuman's model, the ratio Sy/S was required to exceed 10, i.e. the parameter σ ranges between $0 < \sigma = S/Sy \leq 0.1$.



(a) (b) (c) (d)

Figure 3. (a) ultrasonic flow meter; (b) data reading from the TD diver on well B3; (c) piezometer P1 with cap, diver is suspended on cap from inside; (d) drainage of water from B3, level gauge and barrel.

The graphical method includes construction of two graphs in log-log scale, in the same sheet of Excel, with the same number of logarithmic divisions which allows that the main squares of the grid, for both graphs, are of equal dimensions. One graph consists of two families of type curves (Type A and Type B) and the second graph presents time-drawdown test data in scatter form. The graph with the test data is semi-transparent in order that it can be overlaid onto the graph with type curves. Procedure is as follows:

a. Firstly, by moving the graph of the test data, a certain Type B curve is found on which most of the time-drawdown test points from the late period are situated. When a suitable Type B curve is found, the corresponding value of the β parameter is recorded and one common point is selected in the part of the overlapped graphs, which should have integer values on the abscissa and on ordinate, for both graphs. The coordinates of this point are s^* and s_D^* on the ordinate, and t^* and t_y^* on the abscissa. Transmissivity T and specific yield Sy are determined from Eqs. 1 and 2,
b. Then, the test data are shifted horizontally to the left (keeping the same β value) until the early-time-drawdown points coincided with the corresponding Type A curve. A second match point is selected,

with coordinates: s^*, s_D^* , t^* i t_S^*. Transmissivity is computed from Eq. 1, and elastic storage coefficient S from Eq. 3,

c. The procedure is repeated until one pair of Type A and Type B curves is selected that best fits with the test data.

Transmissivity T and specific yield Sy are obtained from coordinates of the late-time match point, using Eq. 1 and Eq. 2, respectively:

$$T = 1/4\pi \cdot (Q \cdot s_D^*/s^*) \quad (1)$$

$$S_y = \left(T \cdot t^*/r^2 \cdot t_y^*\right) \quad (2)$$

Elastic storage coefficient S is obtained from the early-time match point using Eq. 3:

$$S = (T \cdot t^*/r^2 \cdot t_s^*) \quad (3)$$

The additional formation constants of the aquifer are then calculated from the obtained values for T and S using Eqs. 4-7:

$$K_H = T/b \quad (4)$$

$$k_D = \beta \cdot b^2/r^2 \quad (5)$$

$$K_V = k_D \cdot K_H \quad (6)$$

$$S_s = S/b \quad (7)$$

3. Results and discussion

3.1. Processing of data for P1

Type of aquifer is defined by comaparing the shape of time-drawdown curve for the first pumping rate at P1 (Figure 4), with the theoretical time-drawdown relationships for different types of aquifer from literature (Kruseman & de Ridder, 1994). Since the curve has a characteristic "S" shape, it was concluded that the considered aquifer is unconfined aquifer with efect of delayed yield.

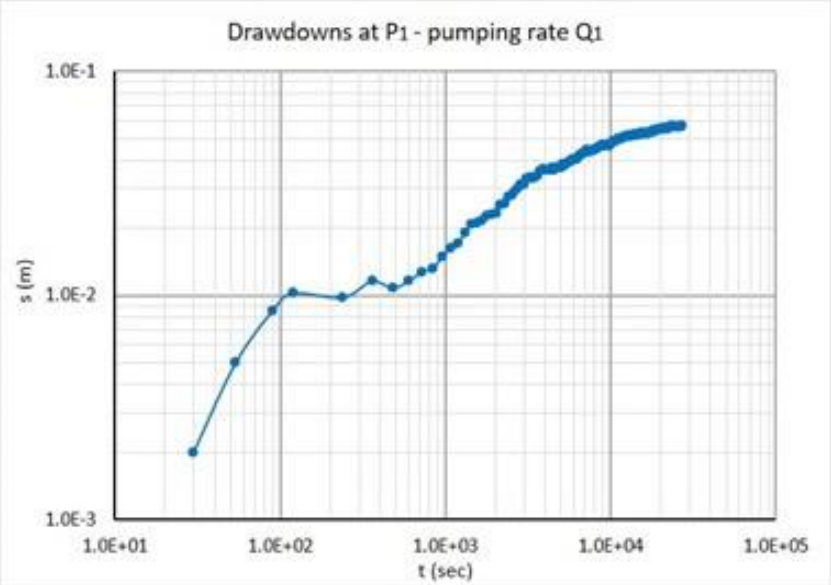


Figure 4. Graph of time-drawdown data for P1 during first flow rate.

For the calculation of formation parameters using the graphical method the following data are used: height of aquifer b = 5.11 m, distance from P1 to B3 r = 7.49 m, flow Q_1 = 1.2 l/s.

In Figures 5 and 6, the two overlaid graphs are presented, one with two families of type curves and the other with time-drawdown data from the test, in the positions that give the best match of the observed data, for the late and early periods, with curves Type A and Type B. Selected pair of type curves has value of β = 0.4.

The common point of the graph for the late test data (green square, Figure 6a) has coordinates: $s_D^* = 4 \cdot 10^{-1}$, $s^* = 6 \cdot 10^{-3}$, $t_y^* = 9 \cdot 10^{-1}$ and $t^* = 1 \cdot 10^3$. Based on these coordinate values, the following formation parameter values are calculated: T = 6.37 × 10⁻³ m²/s and Sy = 0.126. The method demonstrated considerable sensitivity, i.e. a marked variation in the calculated parameters depending on the coordinate values read. If the coordinate values, which are typically read as integer numbers, are increased or decreased by a few numbers of the first decimal place, significantly different parameter values are obtained; for instance, with s_D^*= 4.3 × 10⁻¹ and s^* = 5.8 × 10⁻³, the results are T = 7.08 × 10⁻³ m²/s and Sy = 0.140.

The common point on the plots for early test data (purple square, Figure 6b) has the coordinates: $s_D^* = 4 \cdot 10^0$, $s^* = 6 \cdot 10^{-2}$, $t_s^* = 1 \cdot 10^0$ and $t^* = 1 \cdot 10^2$. Based on these coordinate values, the following aquifer parameter values are determined: T = 6.37·10^{-3} m²/s and S=0.012.

The adopted values are $T = 7.08 \cdot 10^{-3}$ m²/s , $S_y = 0.140$ i $S = 0.013$. Based on the adopted values, additional aquifer parameters are calculated: $K_H = 1.39 \cdot 10^{-3}$ m/s, $k_D = 0.186$, $K_V = 2.58 \cdot 10^{-4}$, $\sigma = 0.09$ i $S_s = 2.47 \cdot 10^{-3}$ 1/m.

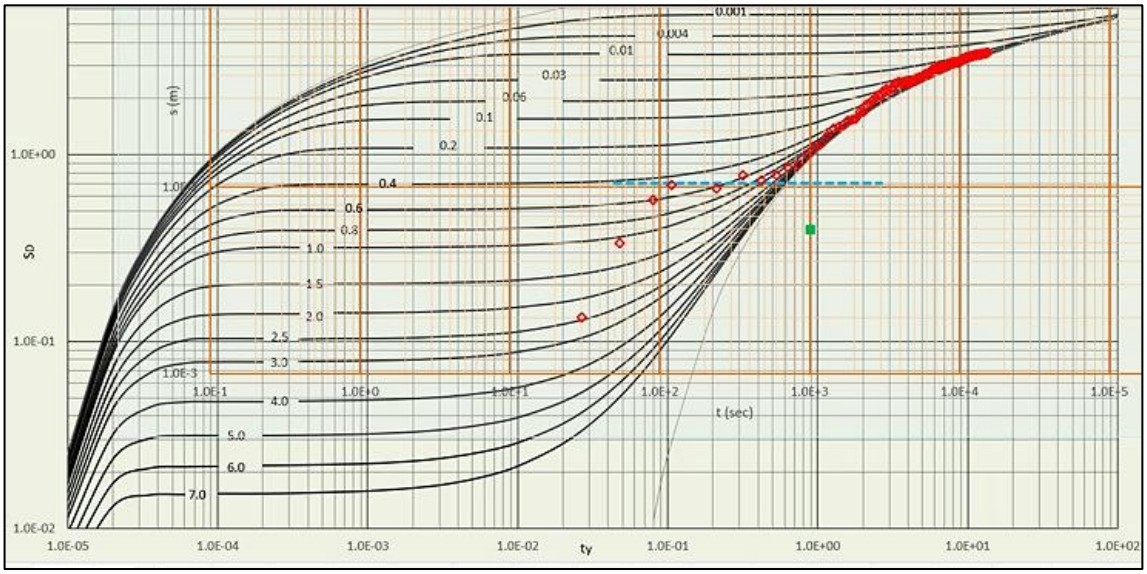


Figure 5. Ilustration of applied graphical method for late test data

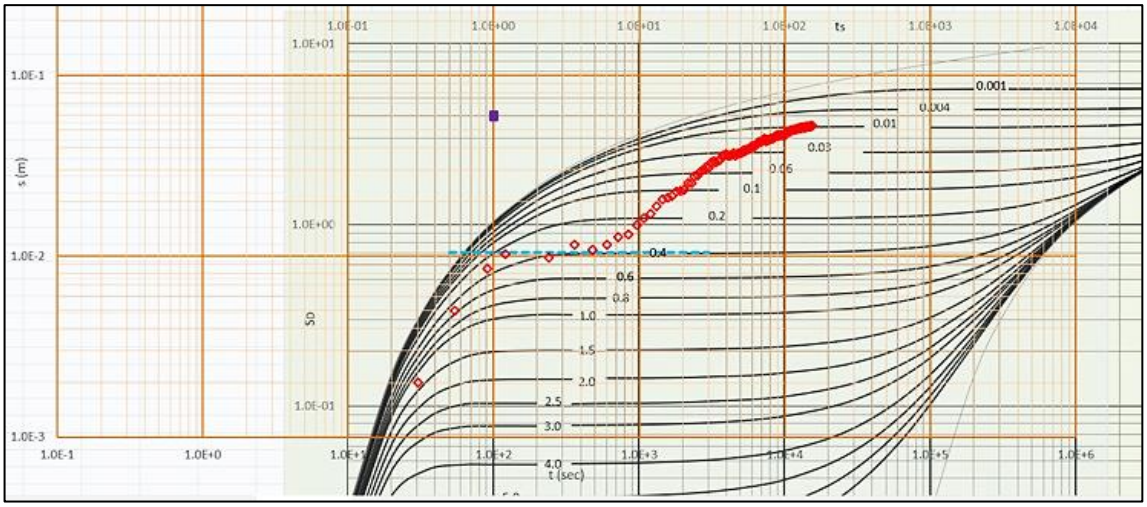


Figure 6. Ilustration of applied graphical method for early test data

Since the graphical method implies processing data from a constant-rate pumping test, whereas the case at hand involves a step-drawdown test with groundwater level recovery monitoring, an "inverse procedure" utilizing a numerical solution of the mathematical model was adopted. As the original numerical solution by Neuman was unavailable to the authors, a simplified numerical model based on the Theis numerical model was developed.

The simplified numerical model includes following changes made in Theis numerical model:
- for data in range $0 < t \leq t_1$=452 min i.e. during first pumping rate Q_1: from 0 to 4 minutes the solution uses S identical to the Theis method; from 4 to 12 minutes the drawdowns remain constant; from 14 to 452 minutes, when pumping at rate Q_2 begins, Sy is used instead of S. In mathematical notation, drawdowns are obtained from Eq. 8,

$$s(r,t) = \frac{Q_1}{4\pi T} \cdot \ln\left[\frac{R_{0,1or0,2}(t)}{r}\right]^2 \tag{8}$$

$$R_{0,1}(t)^2 = \frac{2.25 \cdot T \cdot t}{S} \qquad R_{0,2}(t)^2 = \frac{2.25 \cdot T \cdot t}{S_y}$$

- for data from 452 to 1412 minutes, when pumping at rate Q3 begins, the principle of superposition is applied, using (S + Sy) instead of S for the Q1 and (Q2 - Q_1) terms (Eq. 9),

$$s(r,t) = \frac{Q_1}{4\pi T} \cdot \ln\left[\frac{R_0(t)}{r}\right]^2 + \frac{Q_2 - Q_1}{4\pi T} \cdot \ln\left[\frac{R_1(t)}{r}\right]^2 \tag{9}$$

$$R_0(t)^2 = \frac{2.25 \cdot T \cdot t}{(S + S_y)} \qquad R_1(t)^2 = \frac{2.25 \cdot T \cdot (t - t_1)}{(S + S_y)}$$

- for data from 1412 to 1870 minutes, when the pump is shut off, the principle of superposition is applied, using (S + Sy) instead of S for the Q_1, (Q_2 - Q_1), and (Q_3 – Q_2) terms (Eq. 10),

$$s(r,t) = \frac{Q_1}{4\pi T} \cdot \ln\left[\frac{R_0(t)}{r}\right]^2 + \frac{Q_2 - Q_1}{4\pi T} \cdot \ln\left[\frac{R_1(t)}{r}\right]^2 + \frac{Q_3 - Q_2}{4\pi T} \cdot \ln\left[\frac{R_2(t)}{r}\right]^2 \tag{10}$$

$$R_2(t)^2 = \frac{2.25 \cdot T \cdot (t - t_2)}{(S + S_y)}$$

- for data from 1870 to 2082 min i.e. when modeling the return of the groundwater level, the principle of superposition is used, whereby (S+Sy) is used instead of S with terms Q_1, (Q_2-Q_1) and (Q_3-Q_2), while S is used in addition to term Q_3 identically to Theis (Eq. 11).

$$s(r,t) = \frac{Q_1}{4\pi T} \cdot \ln\left[\frac{R_0(t)}{r}\right]^2 + \frac{Q_2 - Q_1}{4\pi T} \cdot \ln\left[\frac{R_1(t)}{r}\right]^2 + \frac{Q_3 - Q_2}{4\pi T} \cdot \ln\left[\frac{R_2(t)}{r}\right]^2 - \frac{Q_3}{4\pi T} \cdot \ln\left[\frac{R_3(t)}{r}\right]^2 \tag{11}$$

$$R_3(t)^2 = \frac{2.25 \cdot T \cdot (t - t_3)}{S}$$

Figure 7 presents the modeling of drawdown at P1 using the inverse procedure. The pumping rates used were Q_1 = 1.2 l/s, Q_2 = 1.8 l/s, and Q_3 = 2.5 l/s. By applying the numerical model described above, the best fit between the test data and the numerical model data is obtained with the following aquifer parameter values: T = 7.10×10^{-3} m²/s, Sy = 0.125, S = 0.02, and (S + Sy) = 0.145.

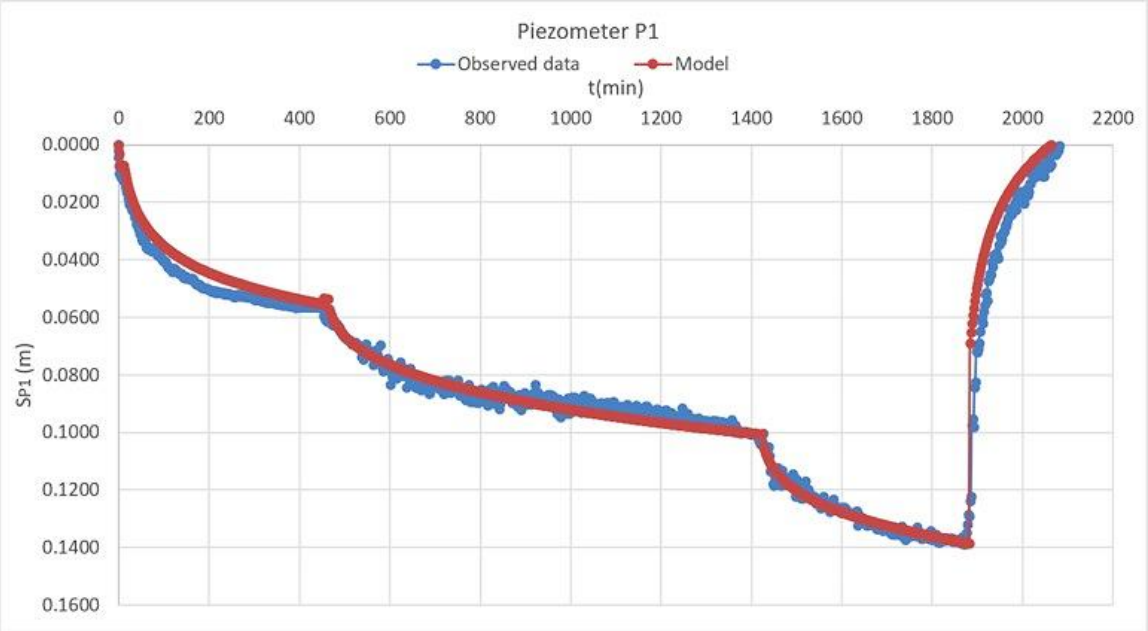


Figure 7. Determination of formation constants using Inverse procedure for P1.

3.2. Processing of data for B3

The first step in the processing of drawdown data for B3 was the determination of parasitic losses: linear character (A_1) and quadratic character (B). Data on the return of groundwater level were extracted from the data set, and the transmissivity value of $T = 7.18 \cdot 10^{-3}$ m2/s was determined by the method of recovery data processing after test with stepwise pumping hydrograph $[s, log\{E(t)\})]$ (Figure 8).

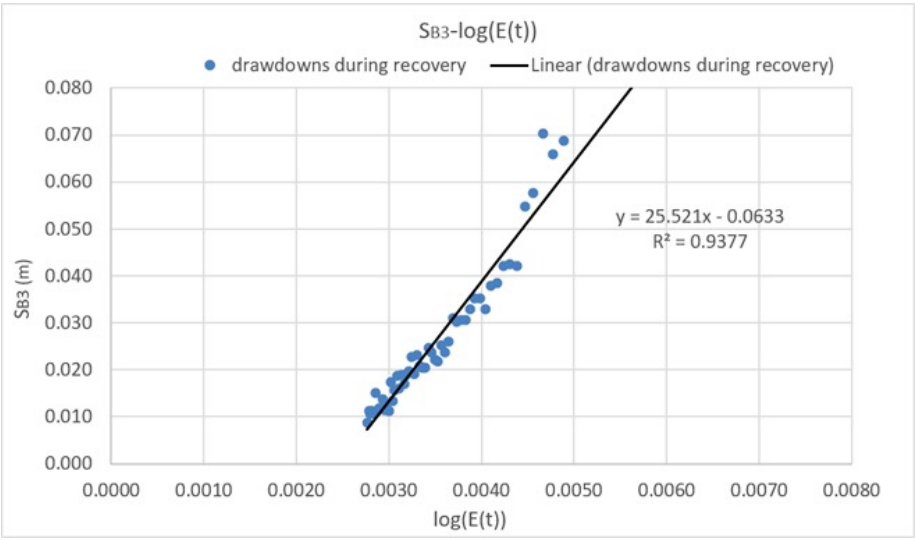


Figure 8. Determination of transmissivity by processing data of groundwater level recovery at B3

Then, the determination of the parasitic loss coefficients at the well is done using method of correcting the drawdowns obtained during test with a stepwise pumping hydrograph. The goal is to obtain drawdowns (s_i^*, where i = 1 ÷ 3) at well B3, which would be obtained if groundwater level measurements, at all three flow rates, are performed at the same radius of influence in well B3.

Figure 9 shows data points (s_i^*/Q_i, Q_i) with a trend line, on the basis of which the parasitic loss coefficients at B3 were determined. The obtained values are: $A_0 + A_1$ = 152.15, and B = 9873. The elastic storage coefficient, determined through data processing at P1, with value of S = 0.02 is adopted, as well as, the

previously determined transmissivity value of $T = 7.18 \cdot 10^{-3}$. With known values of T and S, linear loss coefficient of A_1= 2.6 was determined.

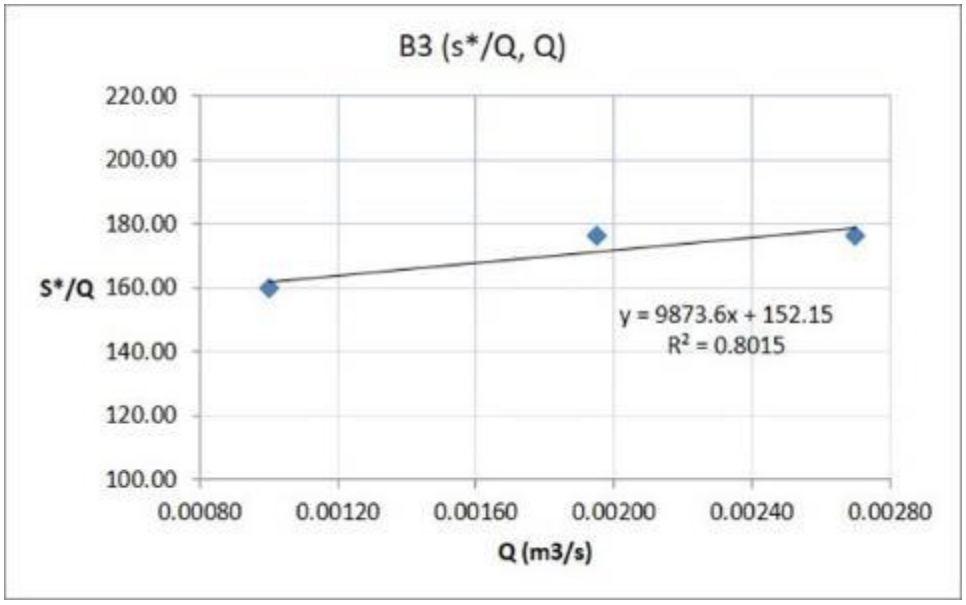


Figure 9. Determination of loss coefficients for well B3

Modeling of drawdown data for B3, observed in the course of test with stepwise pumping hydrograph and recovery, in order to obtain formation constants, was carried out in a similar way as for P1. The difference is that in the case of B3, drawdowns due to parasitic losses at well were also taken into account with the loss coefficients that are tentatively determined by the procedure described above. The best matching of observed and calculated drawdowns was obtained for the following values of flow rate and formation coefficients (Figure 10): Q_1 = 1.00 l/s, Q_2 = 1.95 l/s, Q_3 = 2.7 l/s, A_1 = 2.6, B = 9500, $T = 7.18 \cdot 10^{-3}$ m2/s, S = 0.02, S_y = 0.125, $(S + S_y)$ = 0.145.

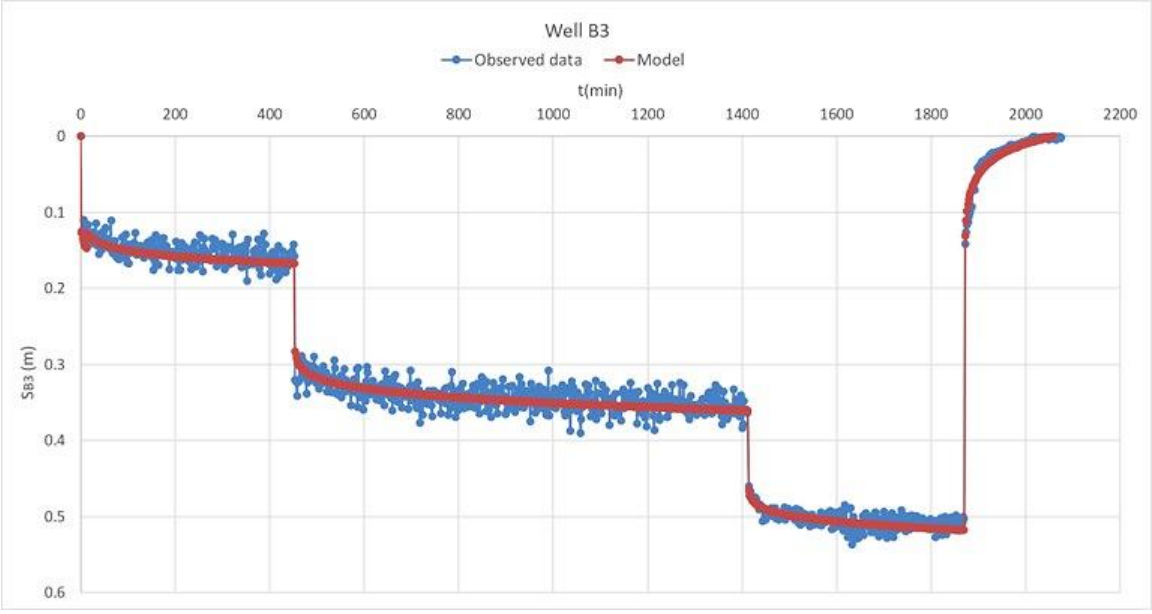


Figure 10. Determination of formation constants using Inverse procedure for P3.

4. Discussion and conclusion

A step drawdown test with three successively increasing pumping rates was carried out at well B3 on the alluvial terrace of the left bank of the Vrbas River at the "University City" site in Banja Luka, with drawdown and recovery monitored at well B3 and piezometer P1. The following conclusions can be drawn:

(1) The time–drawdown curve at piezometer P1 exhibits the characteristic "S" shape that is typical for an unconfined aquifer with delayed gravity yield. Three segments are clearly developed: an early steep segment reflecting release of water from elastic storage by compressibility of the pore water and aquifer skeleton; an intermediate flattening caused by slow gravity drainage above the depression cone; and a late steep segment approaching the Theis response for a confined aquifer. This is the first identification of the aquifer type at this site, and it is consistent with the lithological description of the alluvial sequence (sandy gravels with silty intercalations, hydraulically connected to the Vrbas River).

(2) The formation constants obtained from the combined type-curve and superposition-based interpretation are:
   - transmissivity $T = 7.08–7.20 \times 10^{-3}$ m²/s;
   - specific yield $S_y = 0.125–0.140$;
   - elastic storage coefficient $S = 0.013–0.020$;
   - specific storage $S_S = 2.54–3.91 \times 10^{-3}$ 1/m
   - horizontal hydraulic conductivity $K_H = 1.39–1.41 \times 10^{-3}$ m/s;
   - vertical hydraulic conductivity $K_V = 2.58–2.62 \times 10^{-4}$ m/s;
   - anisotropy ratio $K_H/K_V \approx 5.4$ (or $k_D \approx 0.186$ in the notation of the type-curve parameter);
   - delayed-yield index $1/\alpha \approx 600$ s, with delayed yield ceasing to influence drawdown after approximately 30 min of pumping.

(3) The transmissivity values obtained independently from P1 (type-curve and inverse procedure) and from B3 (recovery analysis and inverse procedure) agree to within 13%, which is satisfactory for a single-well test with a single observation piezometer.

(4) The well-loss coefficients for well B3 were determined as $A_1 = 2.6$ (linear component) and $B = 9500$ (quadratic component). These values should be regarded as preliminary, because they rest on a three-point fit with a coefficient of determination of $R^2 = 0.80$, and because the unit convention used in the specific-capacity plot requires care in application.

(5) The Neuman type-curve method, applied to the first pumping rate, provided preliminary parameter values that were consistent with those from the subsequent superposition-based fit for the full variable-rate drawdown data and recovery data. This demonstrates that a step drawdown test, interpreted with standard type curves and a superposition-based procedure, can yield a consistent set of unconfined-aquifer parameters from one pumping well and a single observation piezometer, without the need for a dedicated constant-rate test. The superposition-based procedure should be regarded as a practical interpretive tool rather than as a new analytical solution.

(6) The parameters reported here are the first baseline formation constants for this stretch of the Vrbas alluvium. They provide the hydraulic input required for the ongoing water-balance assessment at the University City site and can be used as initial values for groundwater-flow modelling and for the design of future pumping tests in the area.

The following limitations should be noted. First, only one observation piezometer (P1) was available in addition to the pumping well, which is sufficient to estimate T, S, and Sy but not to resolve spatial variability or to determine anisotropy independently. Second, the storage parameters used in the B3 fit were adopted from the P1 analysis, so the two fits are not independent with respect to S and Sy. Third, graphical type-curve matching is sensitive to the coordinates of the match points, as demonstrated in this study by the change in T from 6.37 to 7.08×10^{-3} m²/s and in Sy from 0.126 to 0.140 for a small shift in the read coordinates. Fourth, the late-time drawdown data may not have reached complete drainage equilibrium, so the fitted Sy should be considered a lower bound on the true specific yield. Fifth, no prior pumping test exists at this site against which the present results can be cross-checked.

Future work should therefore include: (a) an independent constant-rate pumping test with at least two observation piezometers at different radial distances; (b) independent estimation of specific yield from grain-size analysis or tracer tests; (c) a sensitivity analysis of the fitted parameters with respect to the assumed time at which delayed yield ceases to influence drawdown; and (d) repetition of the test under different seasonal conditions.

Acknowledgement: This paper was supported by the Ministry of Scientific and Technological Development and Higher Education in Republika Srpska, under Grant No. 19.032/961-18/24. The authors gratefully acknowledge the financial support provided.

References

Boulton, N.S. (1963). Analysis of data from non-equilibrium pumping tests allowing for delayed yield from storage. Proceeding of the Institution of civil Engineers, vol. 26, No. 6693.

Gambolati, G. (1976). Transient free surface flow to a well: An analysis of theoretical solutions. Water Resources Research, Vol. 12, pp. 27-39.

Hantush, M.S. & Jacob, C.E. (1955). Non-steady radial flow in an infinite leaky aquifer. Transactions American Geophysical Union , Vol. 36, No. 1.

Kruseman, G.P. & de Ridder, N.A. (1994). Analysis and Evaluation of Pumping Test Data. Sec.Ed., International Institute for Land Reclamation and Improvement, Wageningen, The Netherlands.

Mojičević, M. et. al. (1969). Karta i tumač OGK SFRJ - List Banja Luka. Institut za geološka istraživanja Sarajevo, BiH.

Neuman, S.P. (1975). Analysis of Pumping Test Data From Anisotropic Unconfined aquifers Considering Delayed Gravity Response, Water Resources Research, Vol. 11, No. 2, pp.329-342.

Prickett, T.A. (1965). Type-curve solution to aquifer tests under water-table conditions. Ground Water, 3 (3), 5.

Rorabaugh, M. J. (1953). Graphical and Theoretical Analysis of Step Drawdown Test of Artesian Well. Proceedings of the American Society of Civil Engineers, Vol. 79, No. 362, pp. 1-23.

Streltsova, T.D. (1972). Unconfined aquifer and slow drainage. Journal of Hydrology, Vol. 16, pp. 117-124.

Walton, W.C. (1960). Application and limitation of methods used to analyze pumping test data. Water Well Journal, Feb.-March.